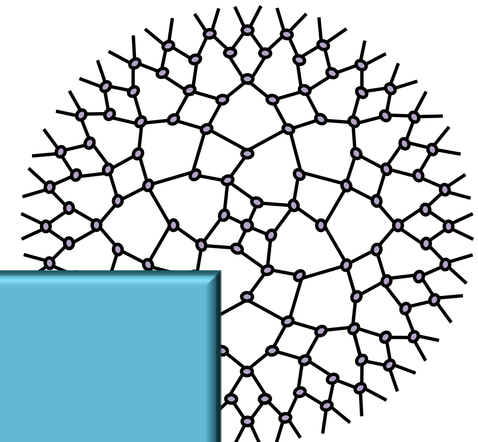
Walter Burke Institute for Theoretical Physics

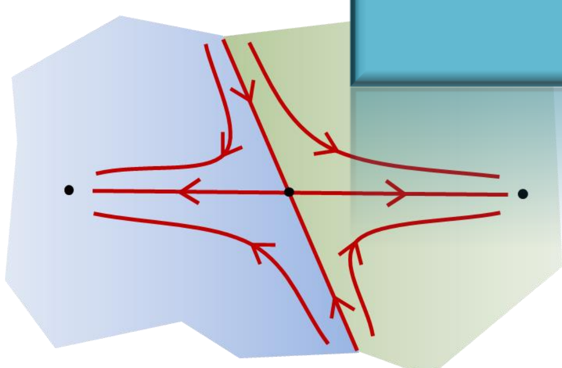
INAUGURAL CELEBRATION AND SYMPOSIUM

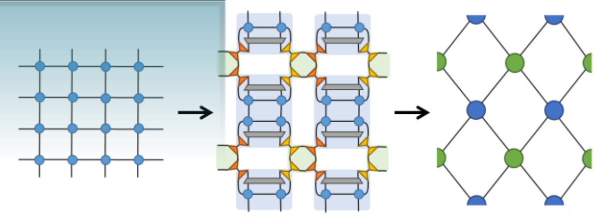
Caltech, Feb 23-24, 2015

$$\int \mathcal{D}\phi e^{-H[\phi,\vec{K}]} = \int \mathcal{D}\phi_{long} e^{-H[\phi_{long},\vec{K}']}$$



Tensor network renormalization





Guifre Vidal

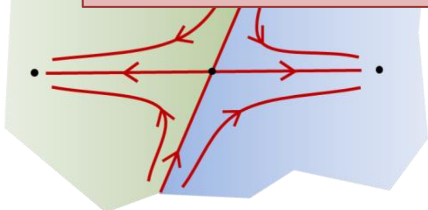
Sherman Fairchild Prize Postdoctoral Fellow (2003-2005)

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

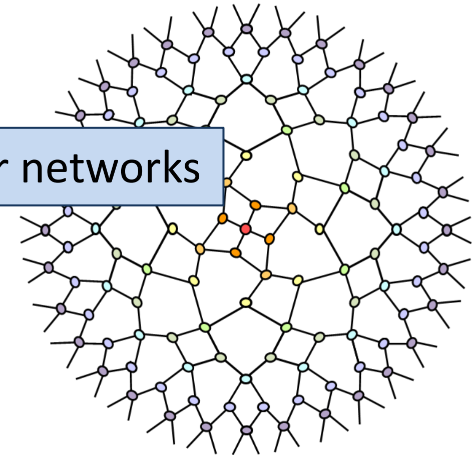
SIMONS FOUNDATION

JOHN TEMPLETON FOUNDATION

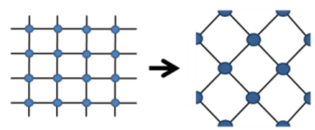
Renormalization group



Tensor networks

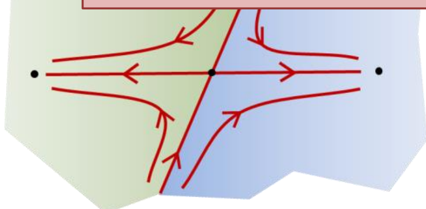


Tensor network renormalization (TNR)

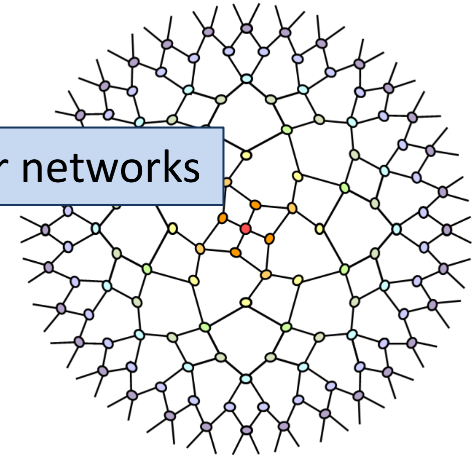


TNR = Renormalization group + Tensor networks

Renormalization group



Tensor networks



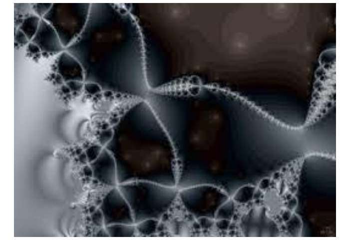
In collaboration with

GLEN EVENBLY

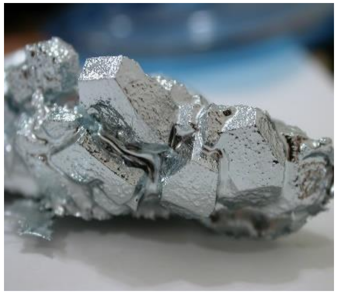
IQIM Caltech → UC Irvine

TNR = Renormalization group + Tensor Networks

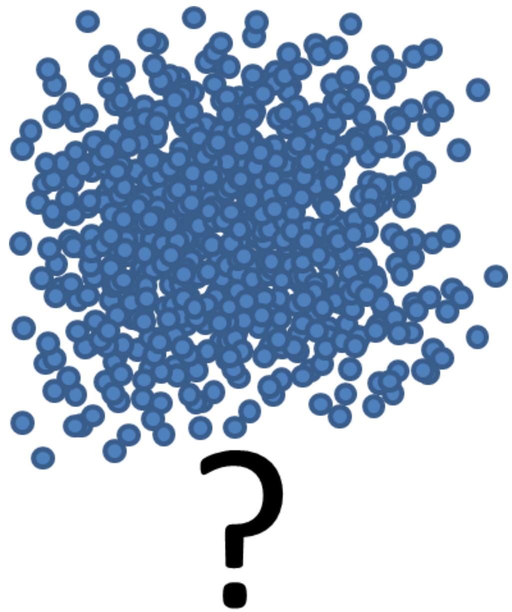
Emergent phenomena in many-body systems

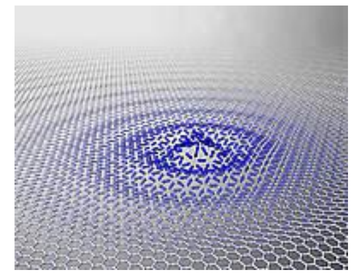


quantum criticality



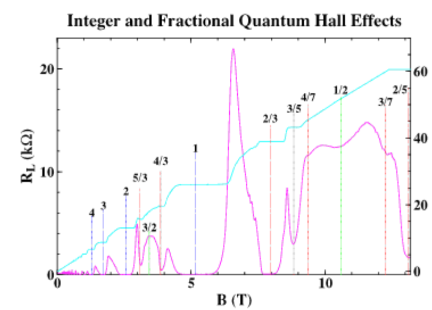
metal





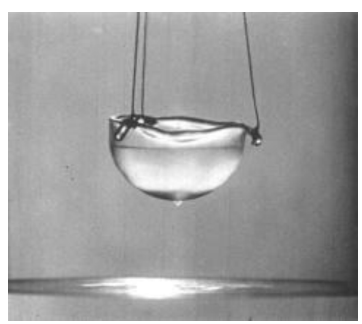
topological order

insulator

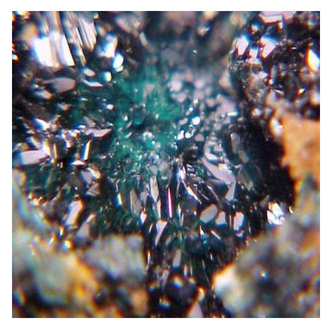


fractional quantum Hall effect

superconductor



superfluid



spin liquid

The problem

we have

we want

H → $|\Psi\rangle$ → $\langle\Psi|o(x,t)o(0,0)|\Psi\rangle$

local Hamiltonian

ground state

low energy,
collective excitations, …

→ $Z = tr\ e^{-\beta H}$

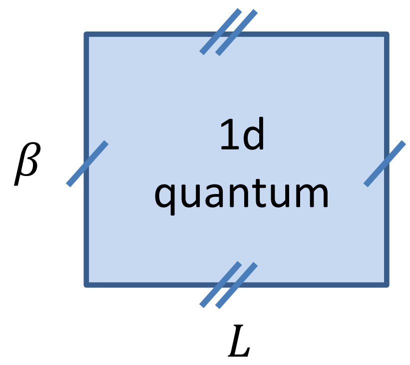
Euclidean path integral

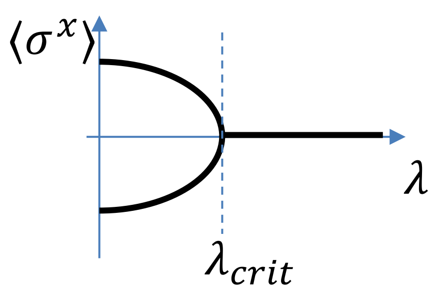
Example:

1d quantum Ising model

$$H_q^{1d} = \sum_i \sigma_i^x \sigma_j^x + \lambda \sum_i \sigma_i^z$$

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



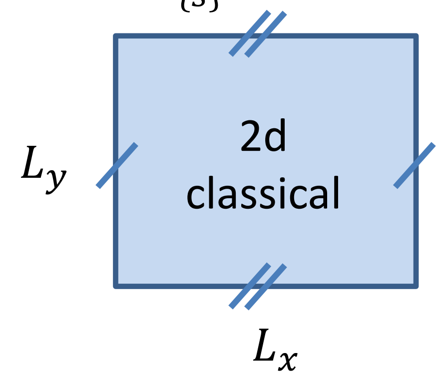


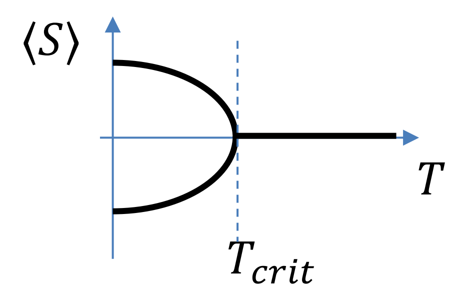
~

2d classical Ising model

$$H_{cl}^{2d} = \sum_{<i,j>} S_i S_j$$

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$





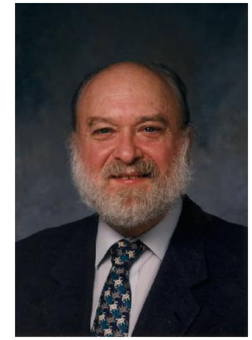
Other examples: material science, quantum chemistry, QCD, …

The Renormalization Group

$$H[\vec{k}] \longrightarrow H[\vec{k}(s)] \qquad \vec{k} = (k_1, k_2, k_3, \cdots)$$

$$H[J, \lambda] = J \sum_i \sigma_i^x \sigma_j^x + \lambda \sum_i \sigma_i^z$$

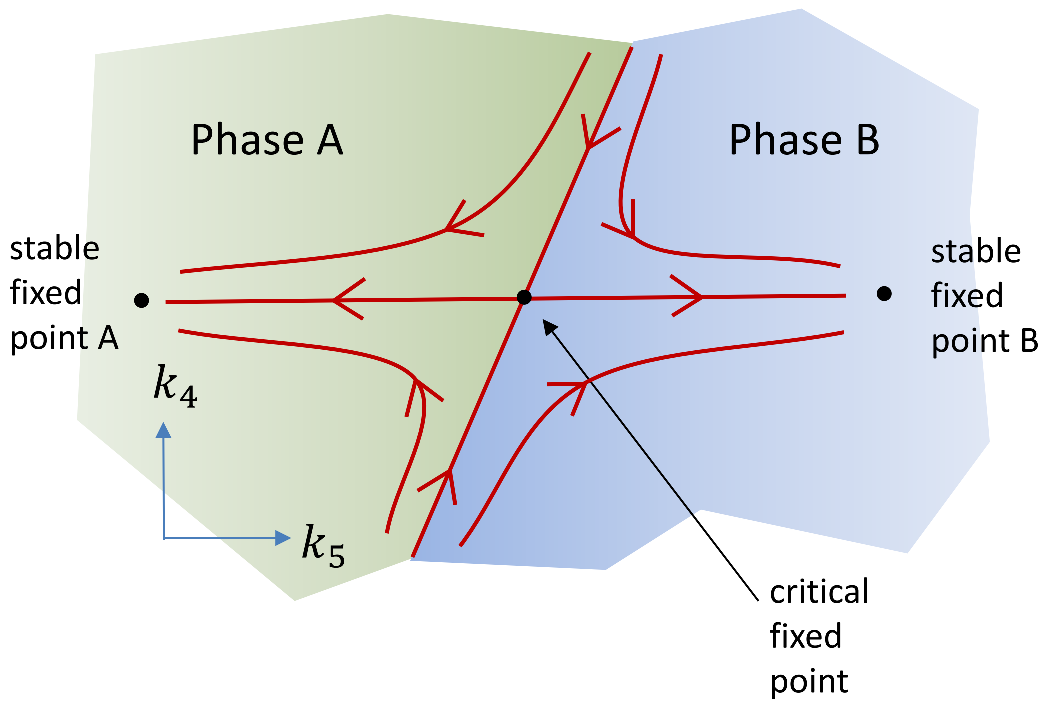
$$H[k_1, k_2, 0, 0, \cdots] = k_1 \sum_i \sigma_i^x \sigma_j^x + k_2 \sum_i \sigma_i^z$$



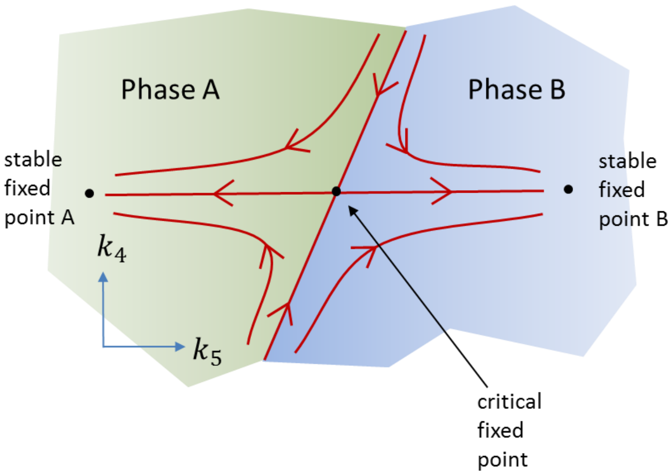
Leo Kadanoff

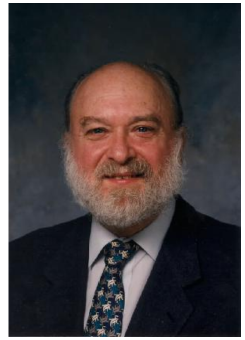
Kenneth Wilson

RG flow in the space of Hamiltonians

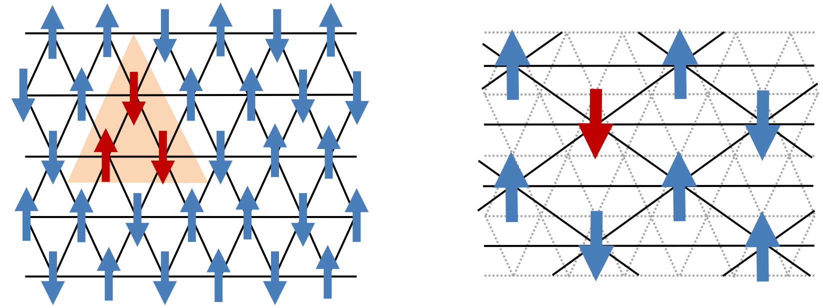


RG flow in the space of Hamiltonians





Leo Kadanof



block spin

+ some rule: majority vote, etc

Change of scale?

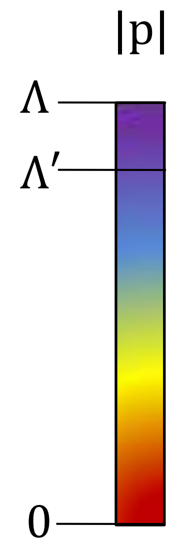
coarse-graining transformation

Kenneth Wilson

$$Z = \int_{|p|\leq\Lambda} \mathcal{D}\phi e^{-H[\phi,\vec{k}]}$$

$$e^{-H[\phi,\vec{k}']} = \int_{\Lambda' \leq |p| \leq \Lambda} \mathcal{D}\phi e^{-H[\phi,\vec{k}]}$$

$$Z = \int_{|p|\geq\Lambda'} \mathcal{D}\phi e^{-H[\phi,\vec{k}']}$$



exact renormalization group equation (ERGE)

We would like (wish list)

- Non-perturbative RG approach

$$H \to H' \to H'' \to \cdots$$

- Universal coarse-graining rules valid for a generic system
- Solve QCD … !

How far did we get ? (over the last 10 years)

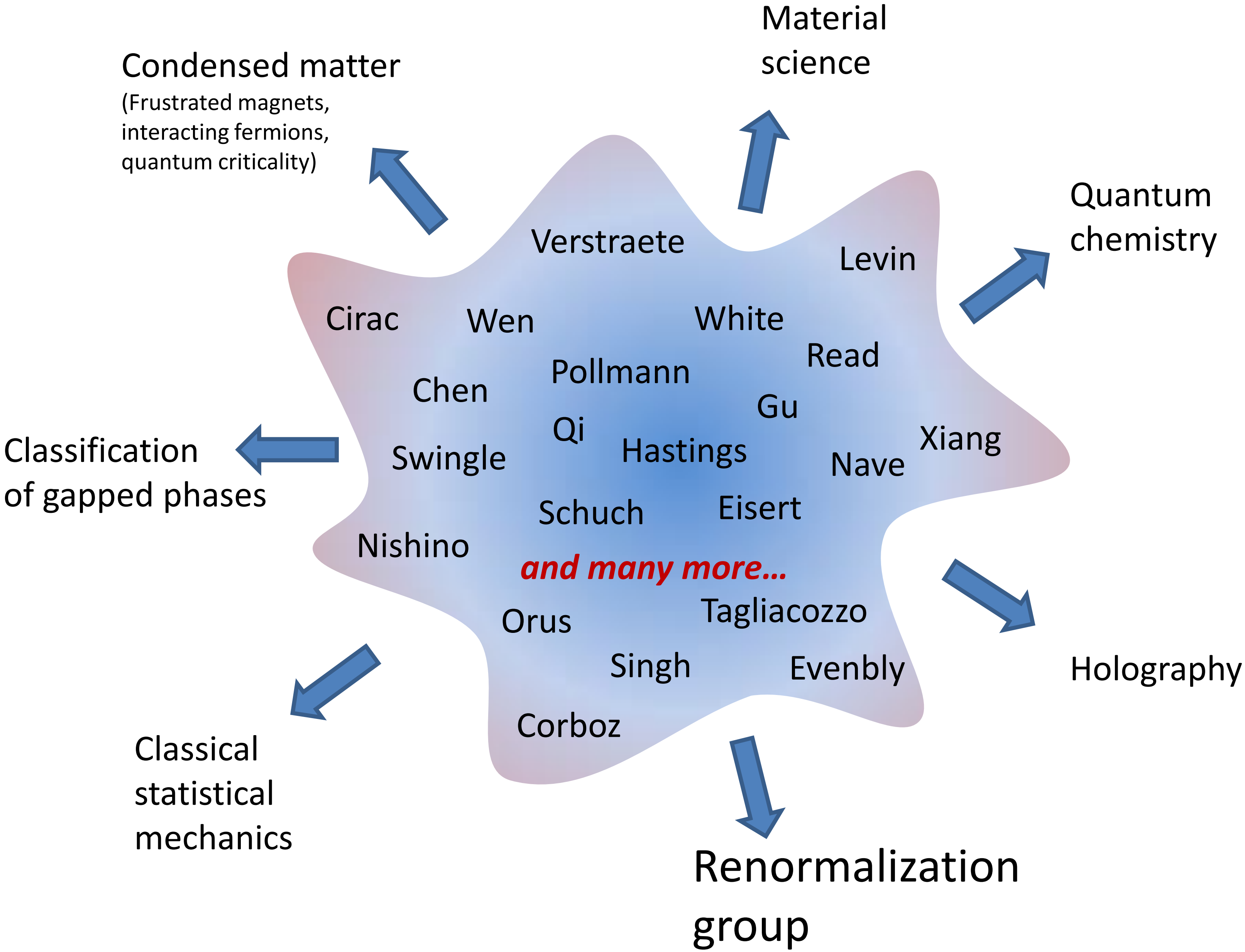
- Reformulated the RG using quantum information tools/concepts (quantum circuits, entanglement)
- Efficient representation of ground state wave-functions $|\Psi\rangle$ (MERA)

universal, non-perturbative, real-space RG approach!

- Key ingredient: **removal of short-rage entanglement**

(but we have not yet solved QCD, sorry…)

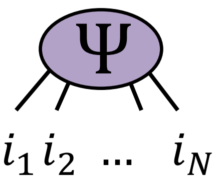
TENSOR NETWORKS



Many-body wave-function of N spins

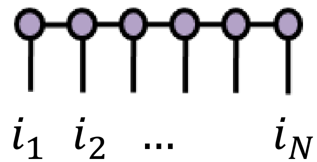
$$|\Psi\rangle = \sum_{i_1, i_2, \cdots, i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N
parameters

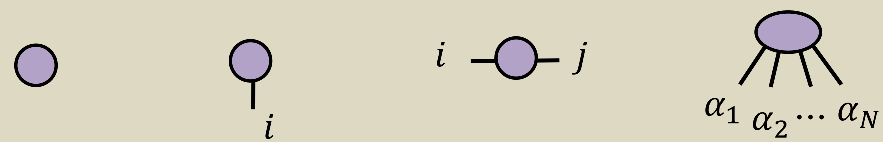


=

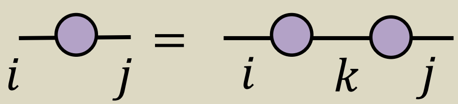
tensor network

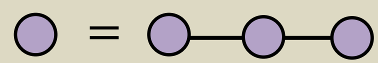


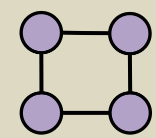
graphical notation



a | b_{i} | c_{ij} | $d_{\alpha_1 \alpha_2 \cdots \alpha_N}$





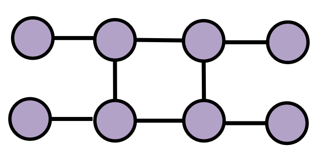


$$T_{ij} = \sum_k R_{ik} S_{kj}$$

$$a = \vec{y}^\dagger \cdot M \cdot \vec{x}$$

$$tr(ABCD)$$

why bother?

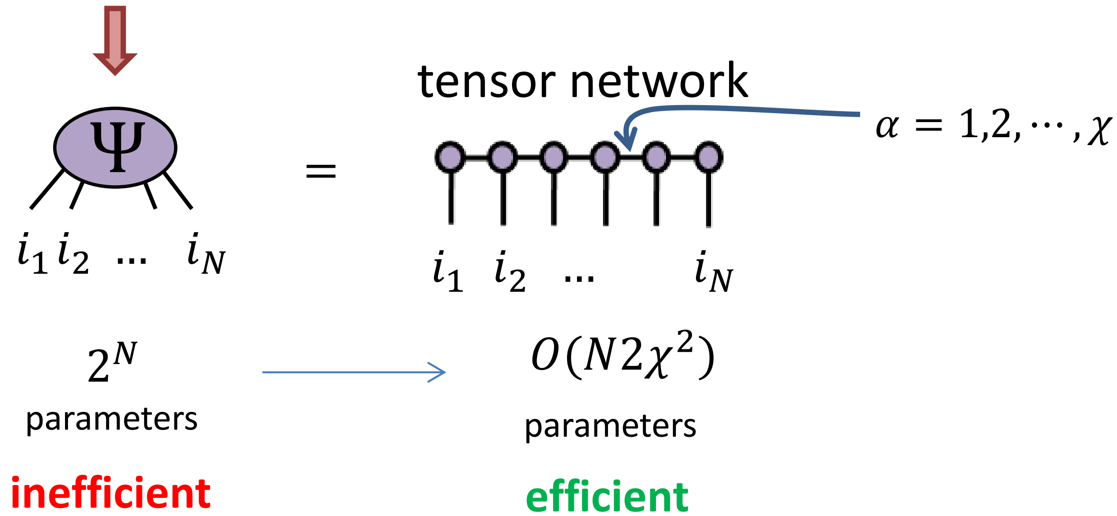


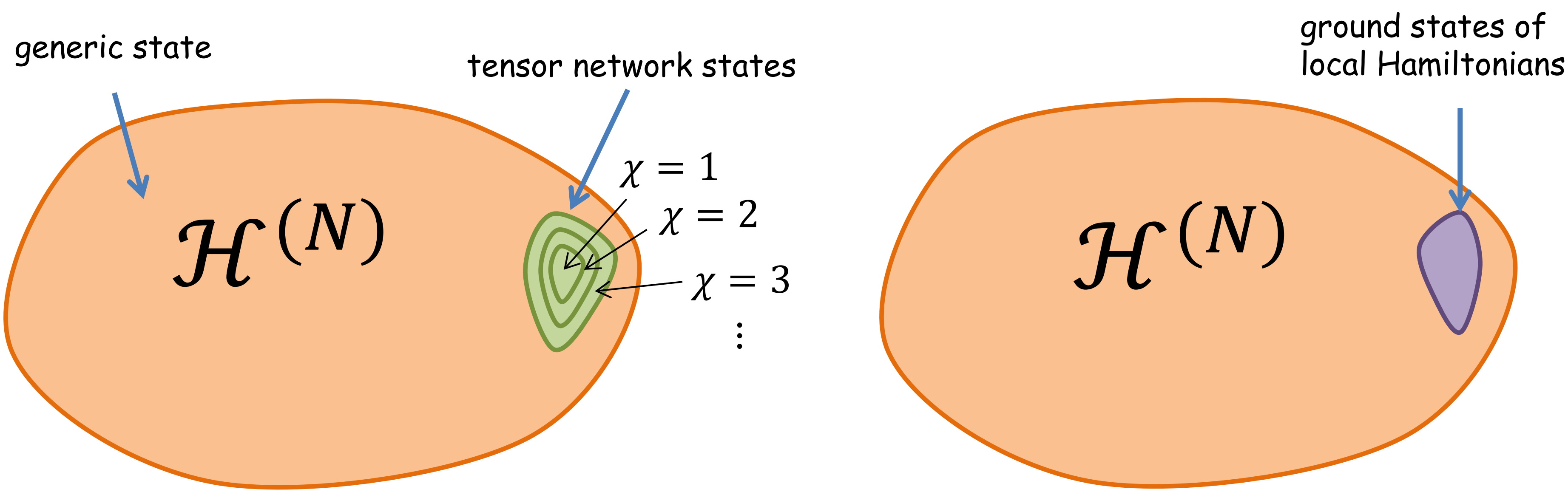
$$\sum_{ijklmnop} A_{ijk} B_{jlm} C_{nko} D_{kmr} x_i y_l z_n v_r$$

Many-body wave-function of N spins

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots,i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

2^N parameters



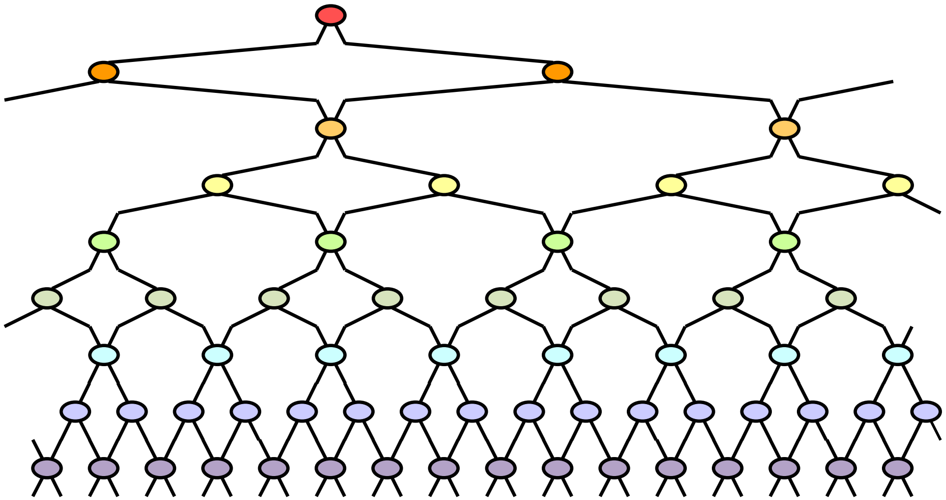


Example of tensor network

Vidal, 2006

Multi-scale entanglement renormalization ansatz

(MERA)

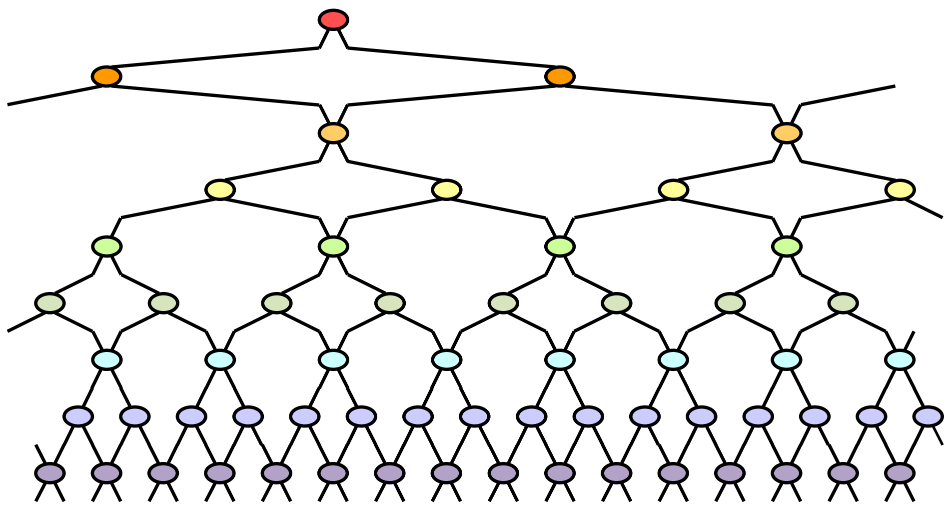


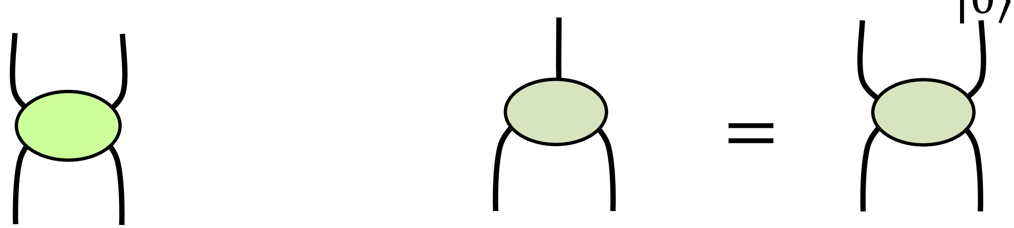
- Variational ansatz for 1d systems, which extends in space and scale
- Variational parameters for different scales
- It is secretly a **quantum circuit** and an **RG transformation**

Example of tensor network

Vidal, 2006

Multi-scale entanglement renormalization ansatz (MERA)





disentangler

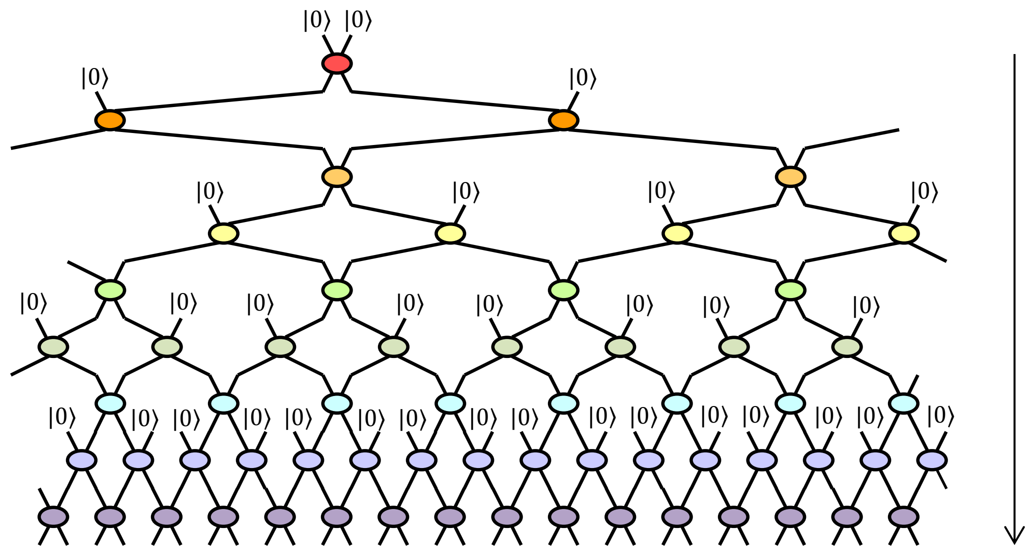
two-body gate

isometry

also a two-body gate

Multi-scale entanglement renormalization ansatz

(MERA)



"time"

quantum circuit U

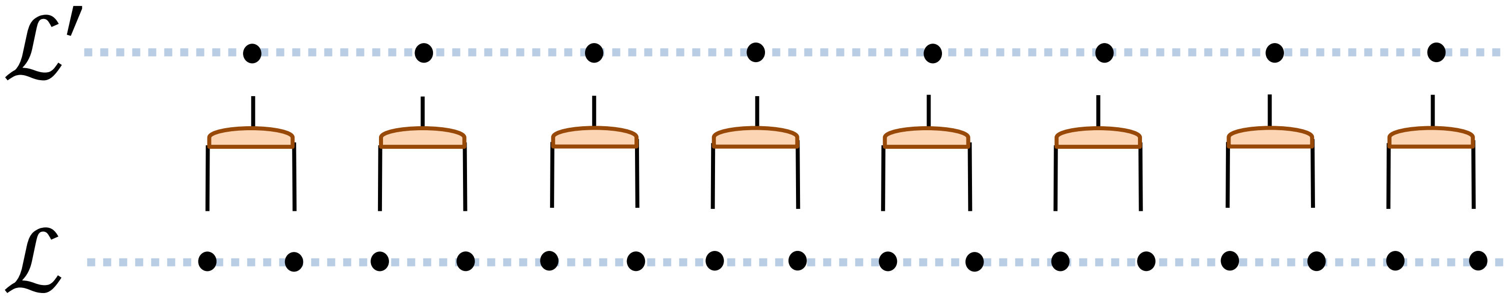
ground state $|\Psi\rangle = U\, |0\rangle \otimes |0\rangle \otimes \cdots \otimes |0\rangle$

Entanglement is introduced by the gates at different times (=scales)

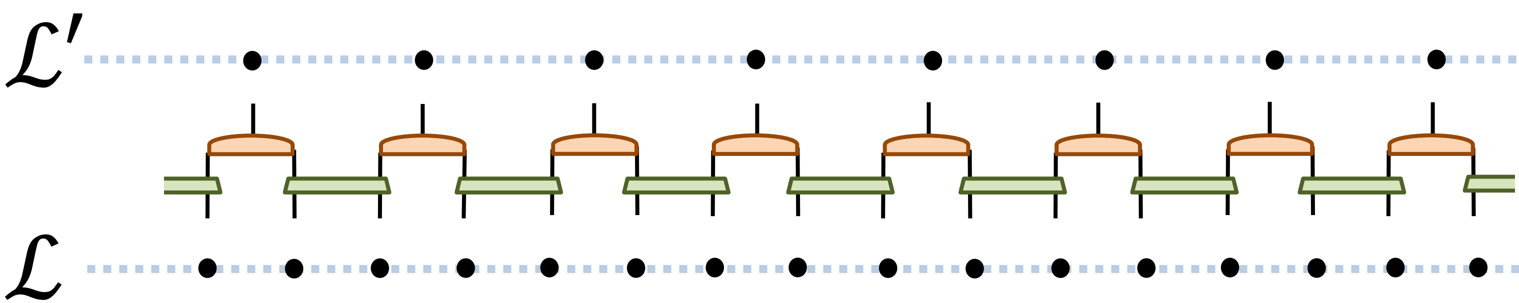
RG Transformation

Kadanoff (1966) + White (1992)

blocking variational optimization



Entanglement renormalization (2005)

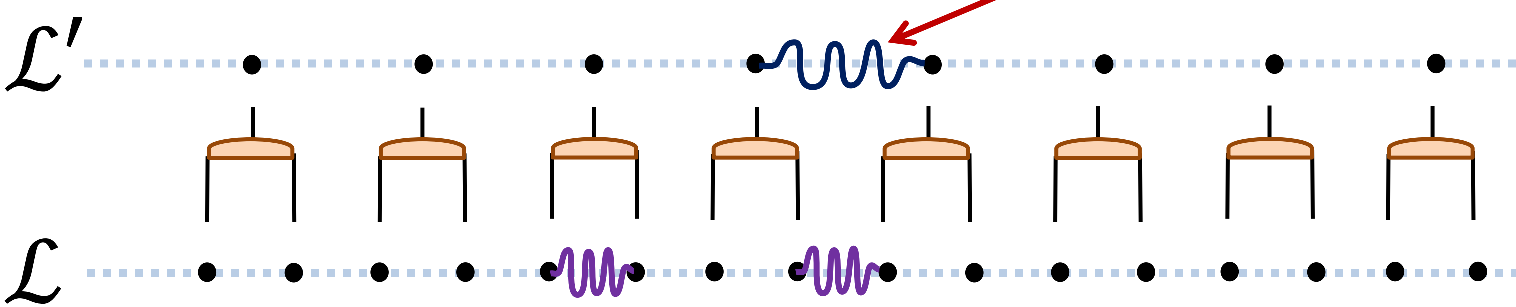


RG Transformation

Kadanoff (1966) + White (1992)

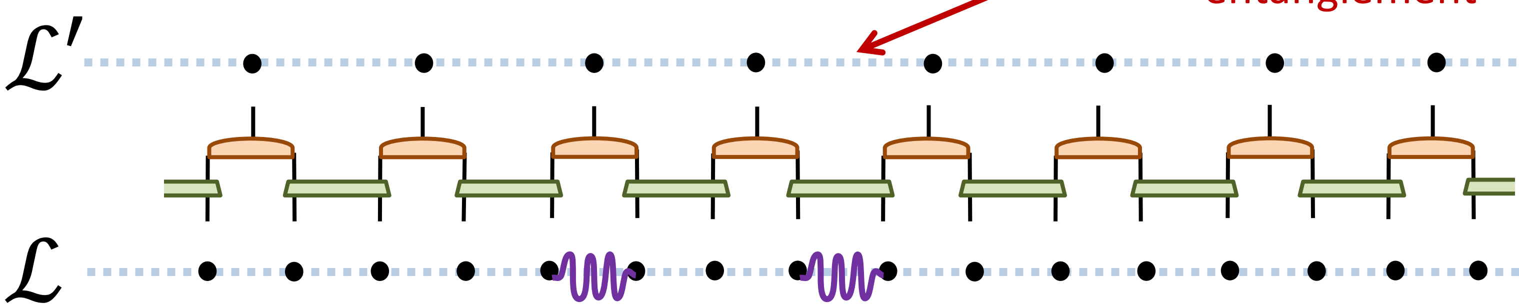
blocking — variational optimization

failure to remove ***some*** short-range entanglement !

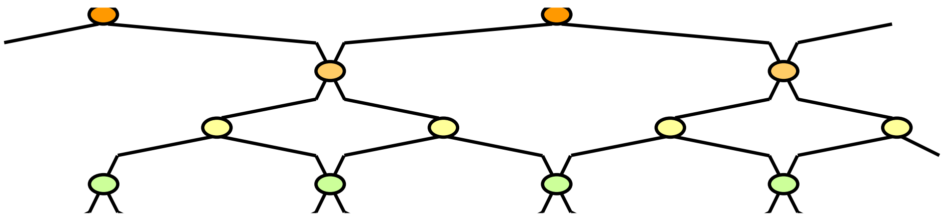


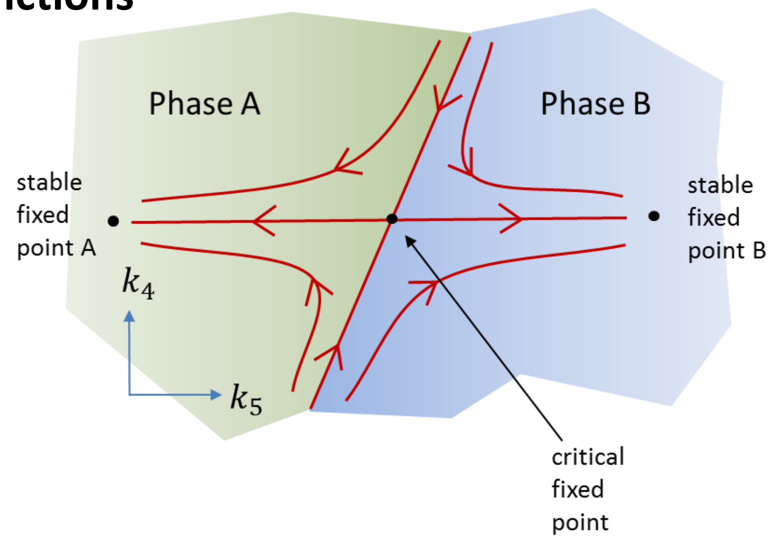
Entanglement renormalization (2005)

removal of ***all*** short-range entanglement



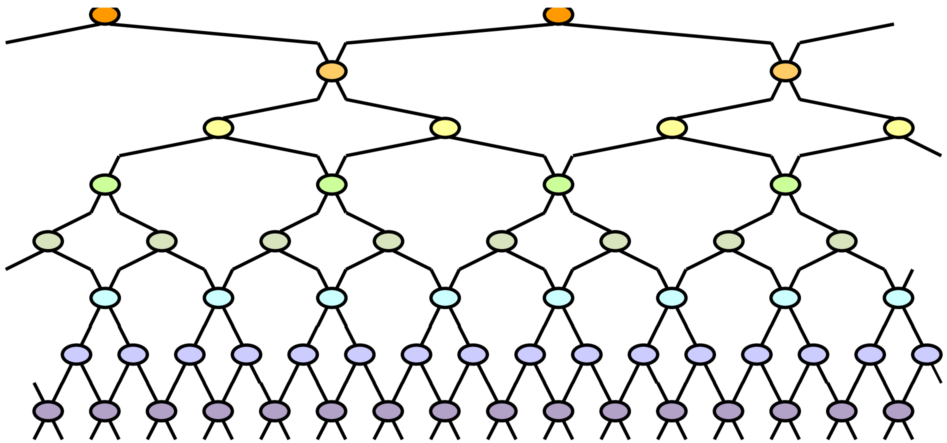
MERA -> RG flow in the space of ground state **wave-functions**

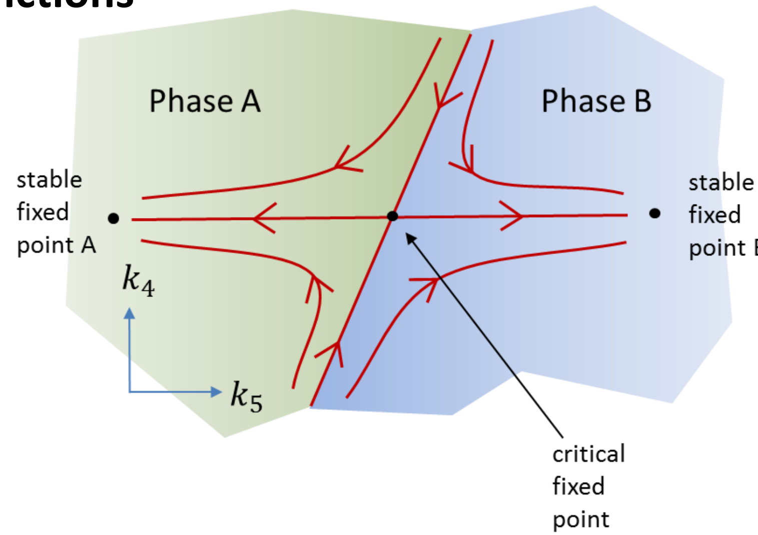




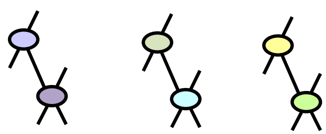
$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots \to |\Psi^{fp}\rangle$$

MERA -> RG flow in the space of ground state **wave-functions**





$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots \rightarrow |\Psi^{fp}\rangle$$

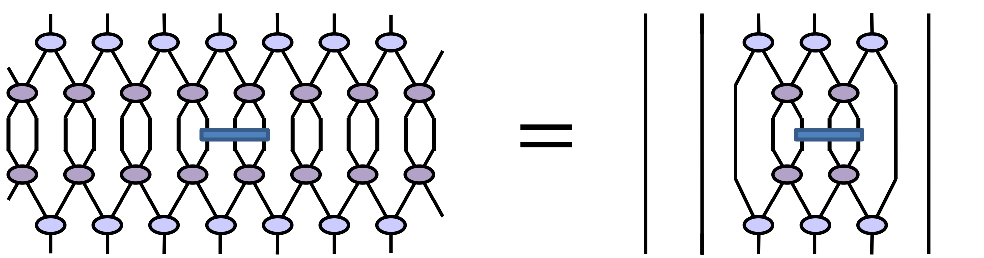


fixed-point wave-function

- topological order (2+1)
- quantum criticality (1+1)

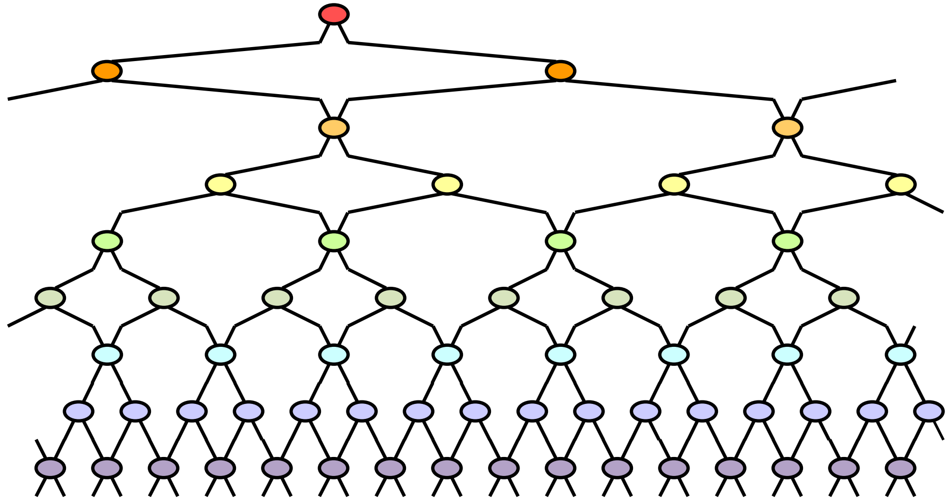
MERA -> RG flow in the space of **Hamiltonians**

$$H \rightarrow H' \rightarrow H'' \rightarrow \cdots \rightarrow H^{fp}$$



local operators are mapped into local operators !

Summary so far



MERA

- Variational parameters for different length scales

- It is secretly a **quantum circuit** → *"entanglement at different length scales"*

  and an **RG transformation**

"removes short-range entanglement" $|\Psi\rangle \to |\Psi'\rangle$ $H \to H'$ *"preservation of locality"*

- blah, blah, blah...

Does it work?

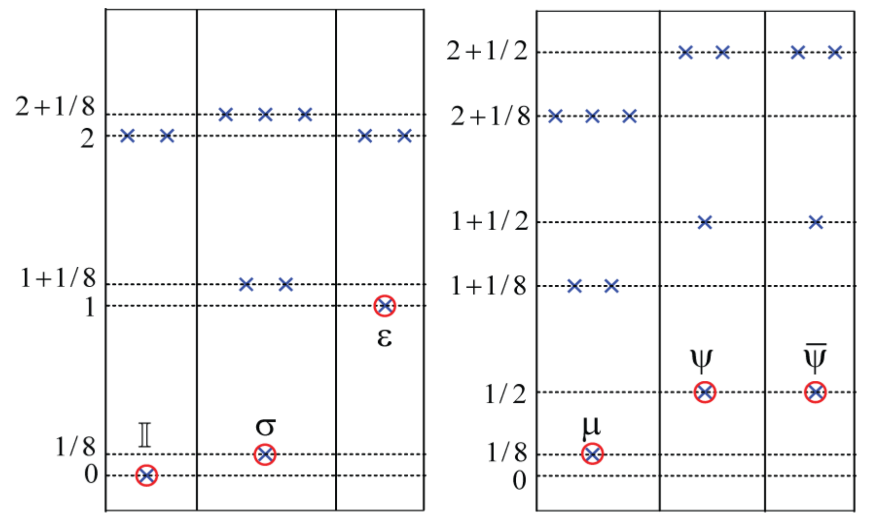
- Optimize variational parameters by energy minimization

Example: Critical quantum Ising model

Pfeifer, Evenbly, Vidal (2008)

Scaling dimensions of primary fields

		scaling dimension (exact)	scaling dimension (MERA)	error
identity	$\mathbb{I}$	0	0	----
spin	σ	0.125	0.124997	0.003%
energy density	ε	1	0.99993	0.007%
disorder	μ	0.125	0.1250002	0.0002%
fermions	ψ	0.5	0.5	$<10^{-8}$ %
	$\bar{\psi}$	0.5	0.5	$<10^{-8}$ %



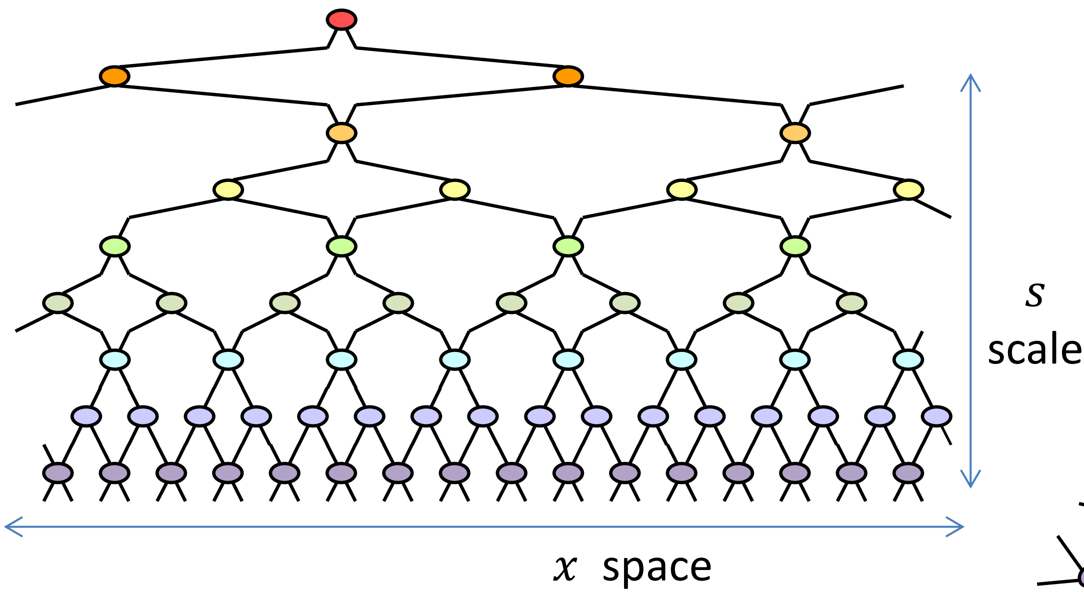
Operator product expansion (OPE) coefficients

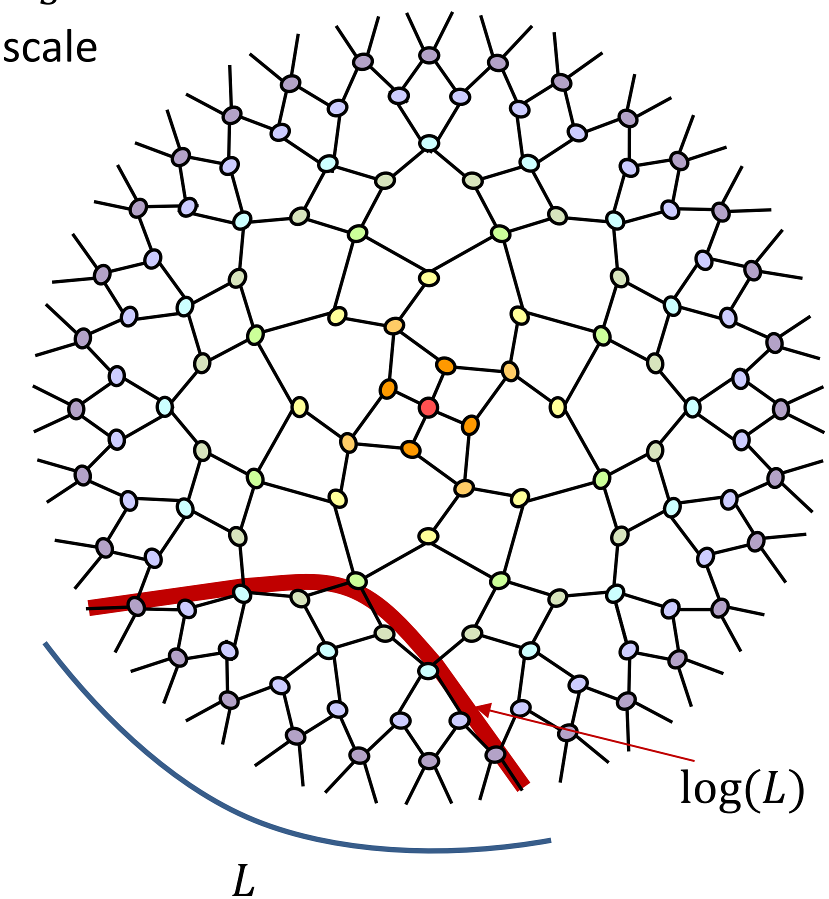
$$C_{\epsilon\sigma\sigma} = \frac{1}{2} \qquad C_{\epsilon\mu\mu} = \frac{-1}{2} \qquad C_{\epsilon\psi\bar{\psi}} = i \qquad C_{\epsilon\bar{\psi}\psi} = -i \qquad C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}} \qquad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}} \qquad (\pm 6\times 10^{-4})$$

scale-invariant MERA
→ conformal data of a CFT:

central charge c
scaling dimensions $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
conformal spin $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
OPE $C_{\alpha\beta\gamma}$

MERA and HOLOGRAPHY





- entanglement entropy

$$S_L \approx \log(L)$$

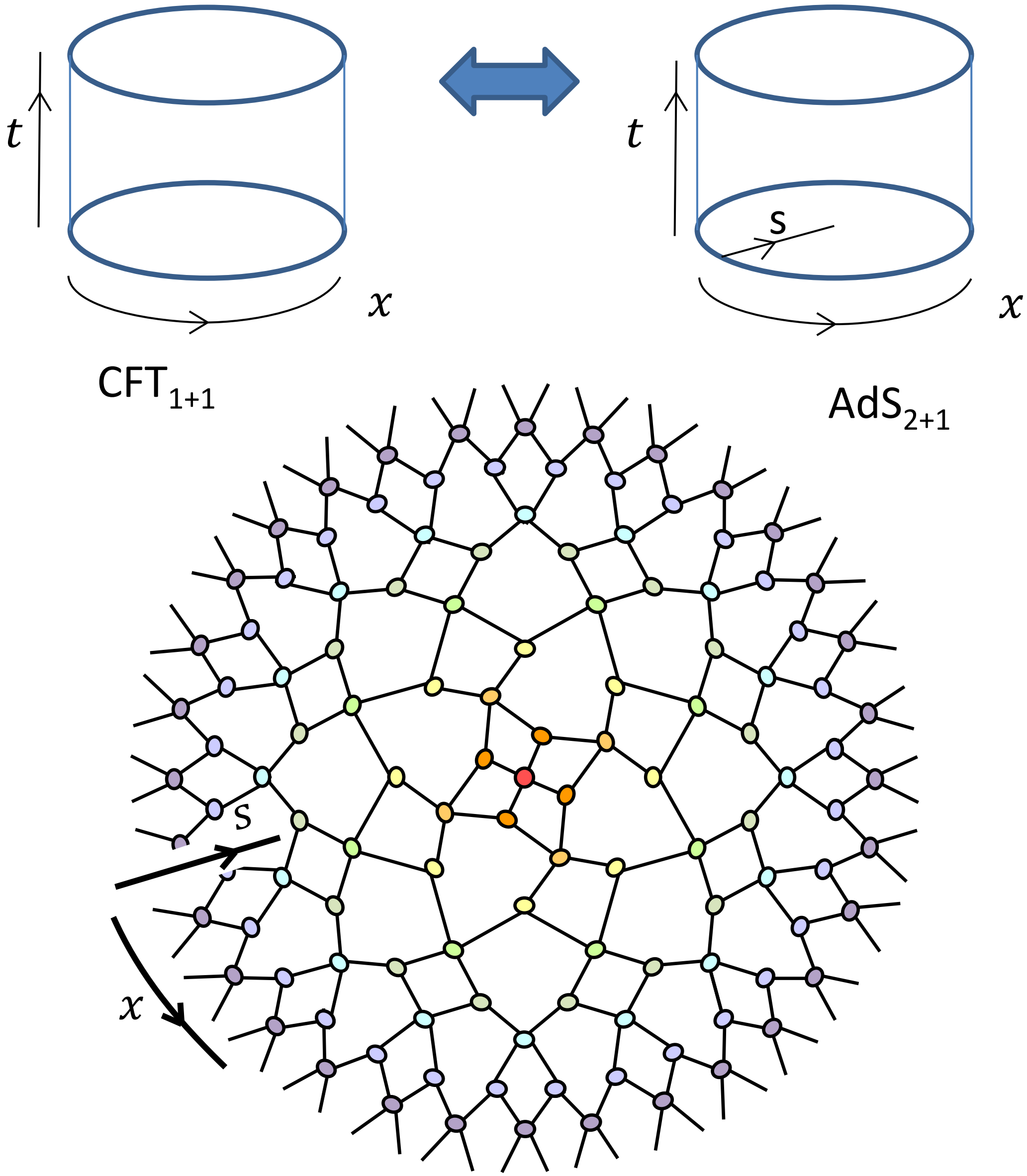
parallel to area of minimal surface in Ryu-Takayanagi

- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

geodesic distance $D \approx \log(L)$ as in hyperbolic space

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



Swingle 2009

Qi

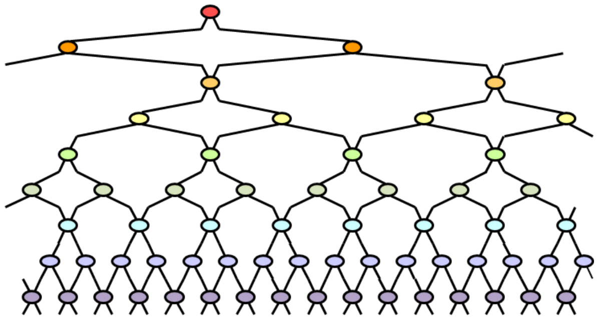
Hartman, Maldacena

Haegeman, Osborne,
Verschelde, Verstraete

Ryu, Takayanagi

Sully, Czech

Harlow, Yoshida, Pastawki, Preskill



So, MERA seems to work!

Great! However

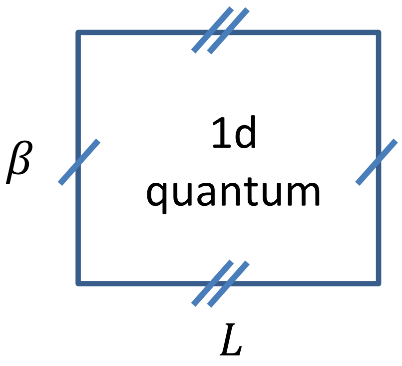
- variational optimization is expensive; local minima.
- do we get the correct ground state?
- Euclidean path integrals / classical partition functions?

Tensor Network Renormalization

Evenbly, Vidal 2014-2015

Euclidean path integral

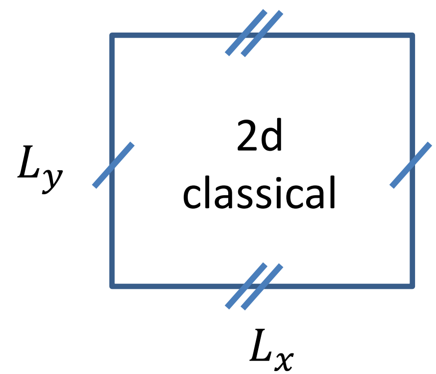
$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$



~

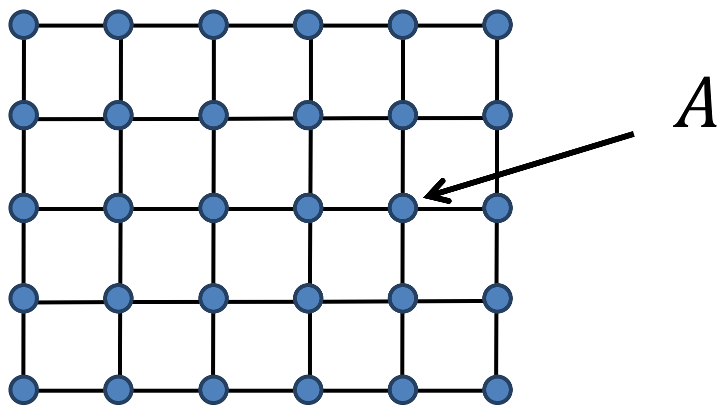
Statistical partition function

$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$



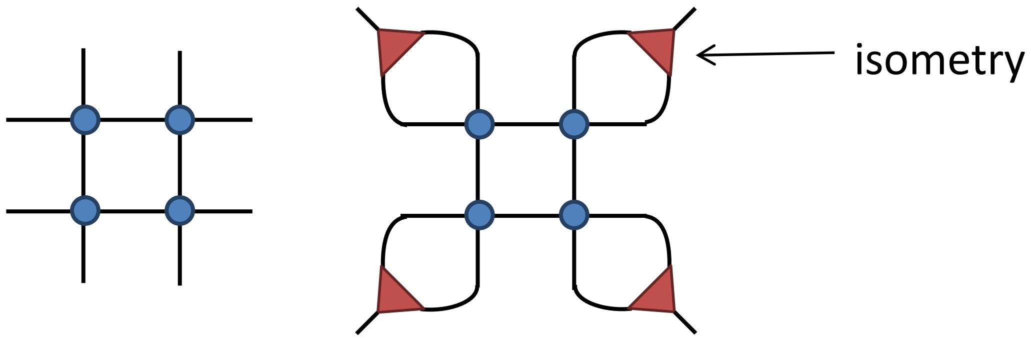
as a tensor network

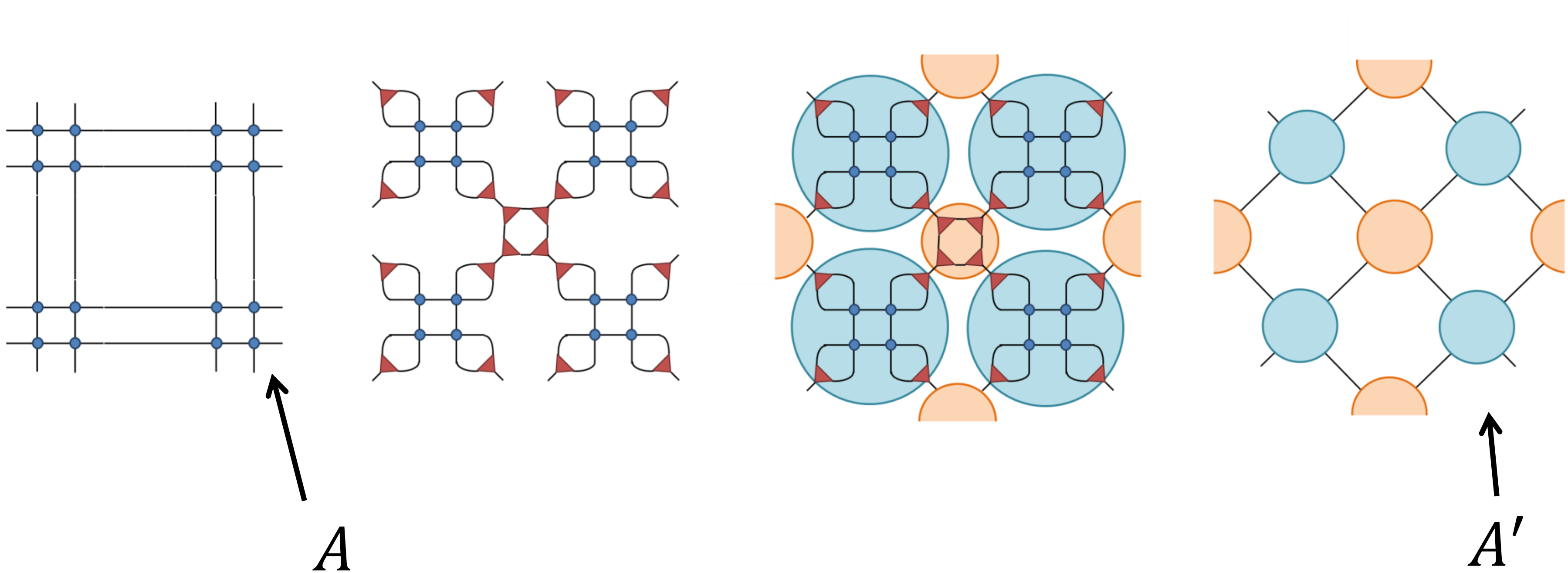
$Z =$



Tensor Renormalization Group (TRG)

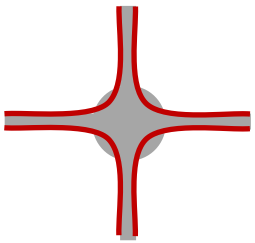
Levin, Nave 2006



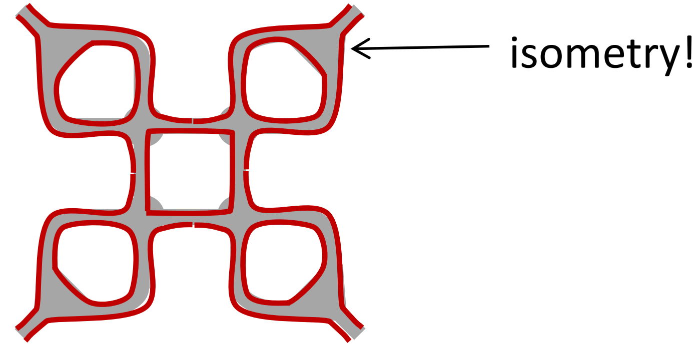


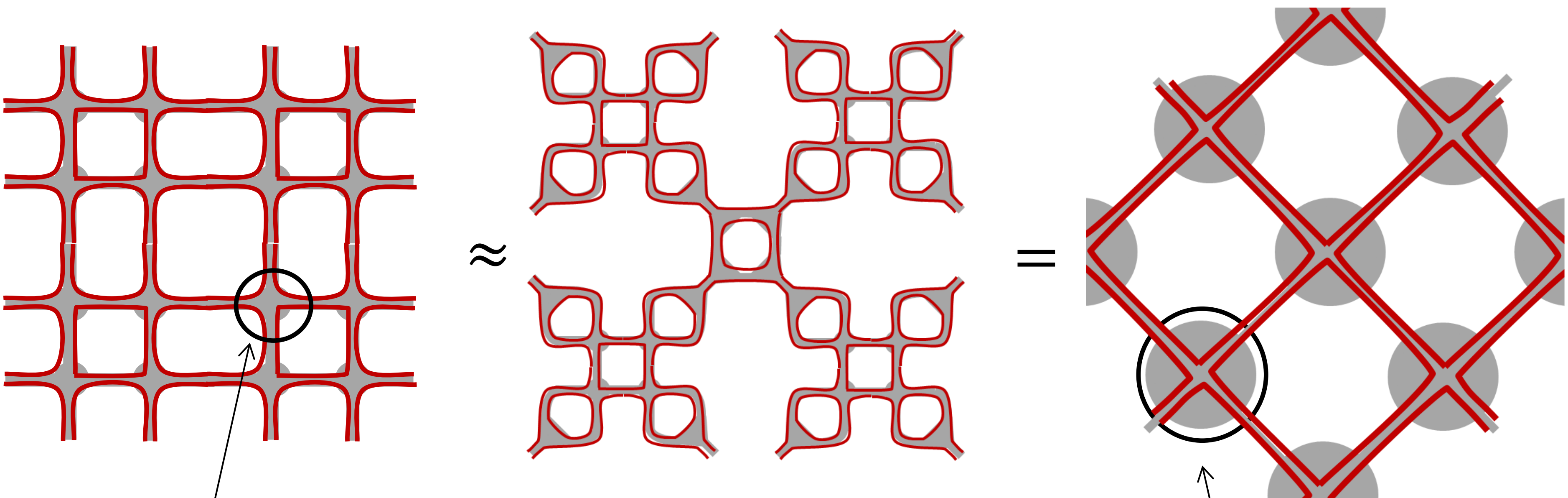
Fixed-point of TRG

Levin, Nave 2006



CDL Tensor
(zero correlation length)



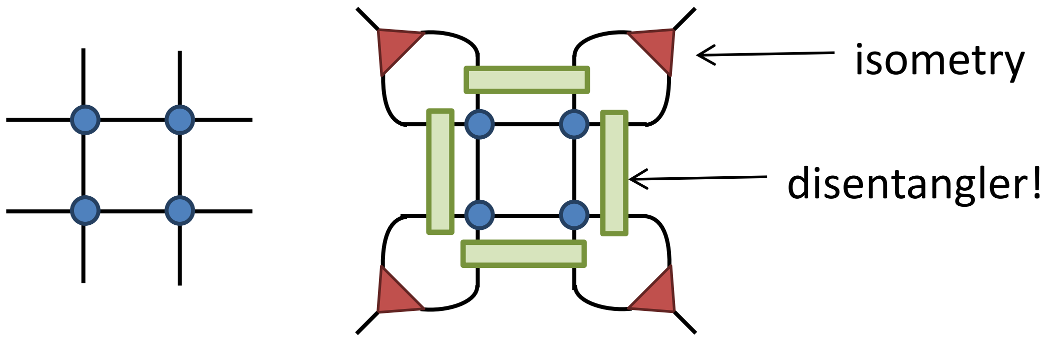


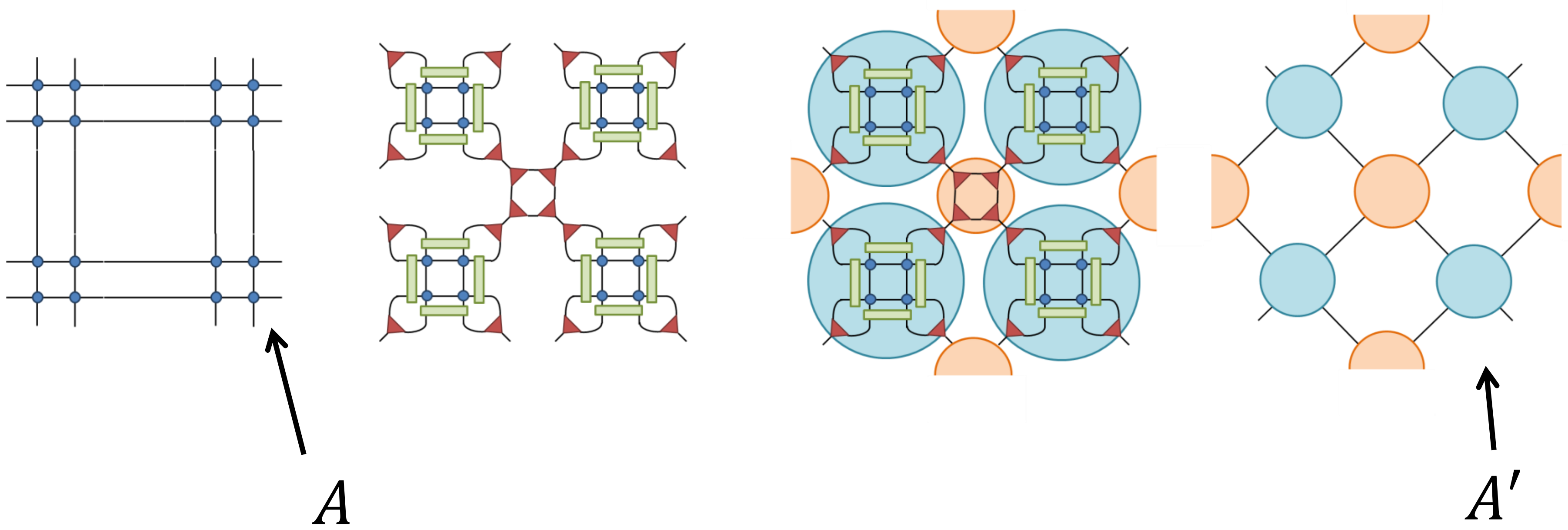
A $\qquad$ A'

some short-range entanglement
has not been removed

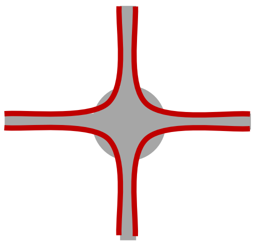
Tensor Network Renormalization (TNR)

Evenbly, Vidal 2014-2015

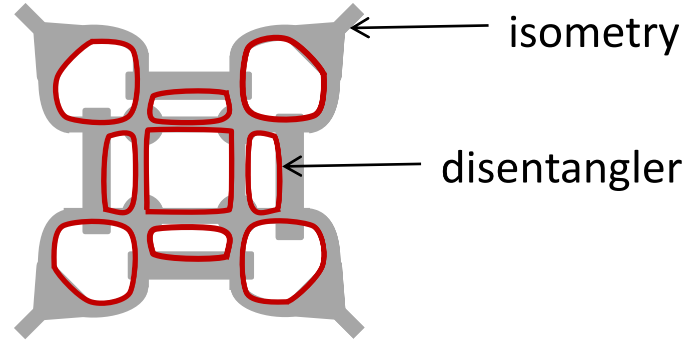


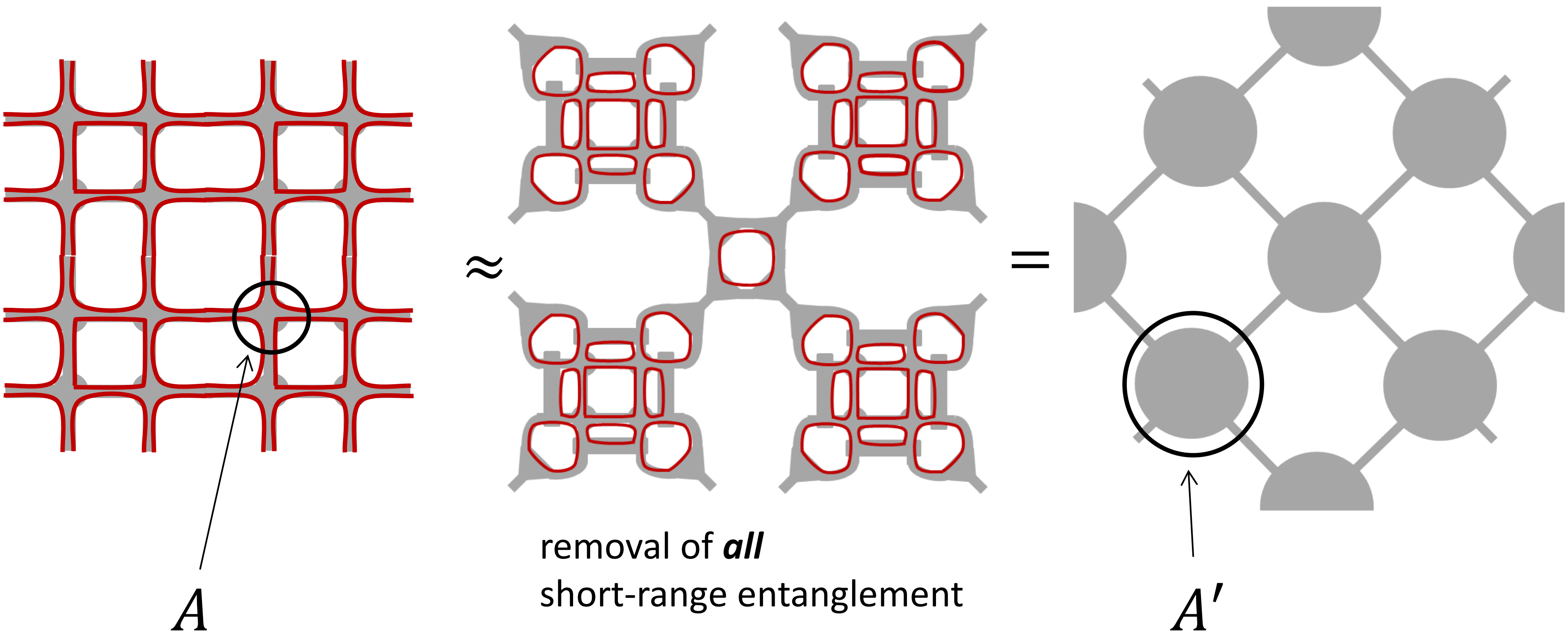


Tensor Network Renormalization (TNR)



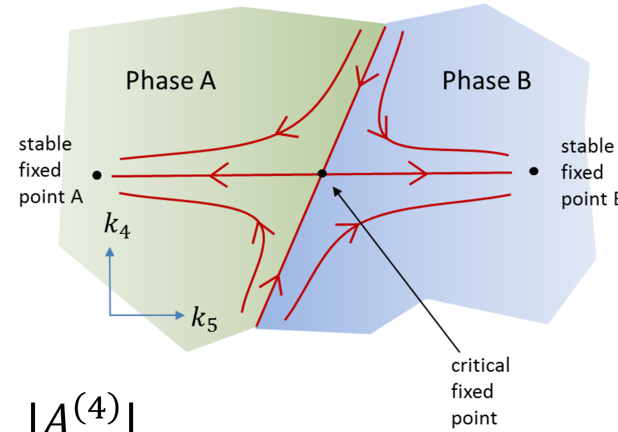
CDL Tensor
(zero correlation length)





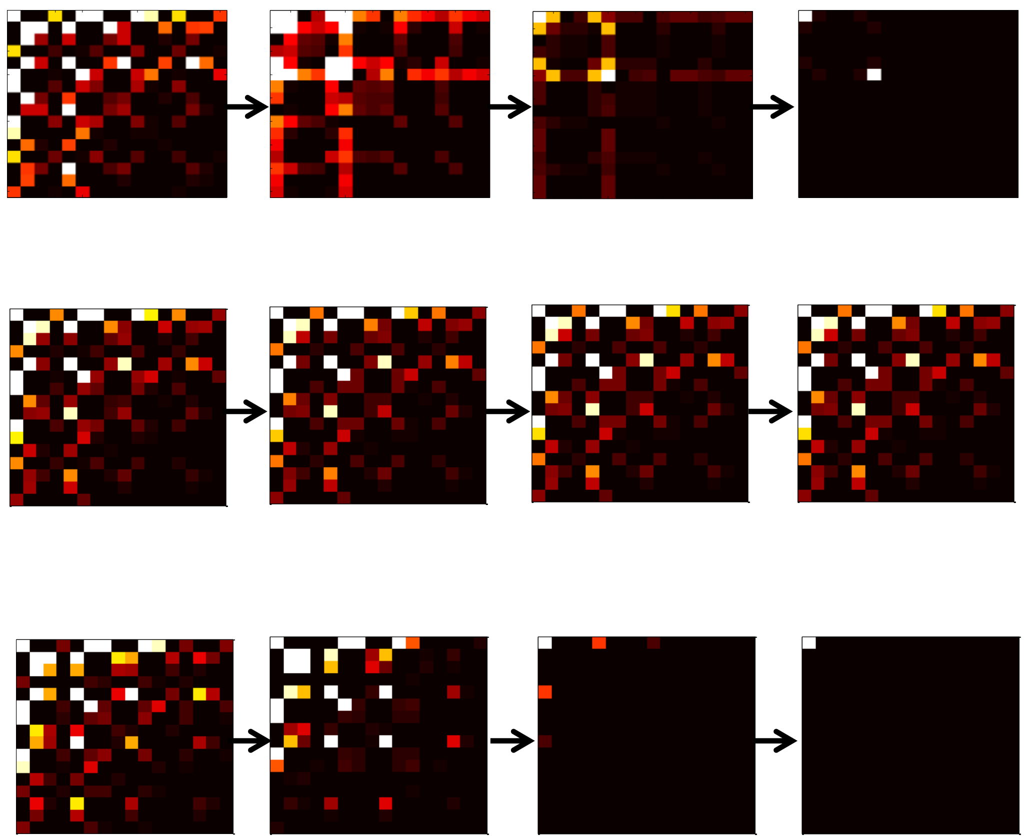
[for CDL tensors, see also Gu and Wen 2009 Tensor Entanglement Filtering Renormalization (TEFR)]

TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$



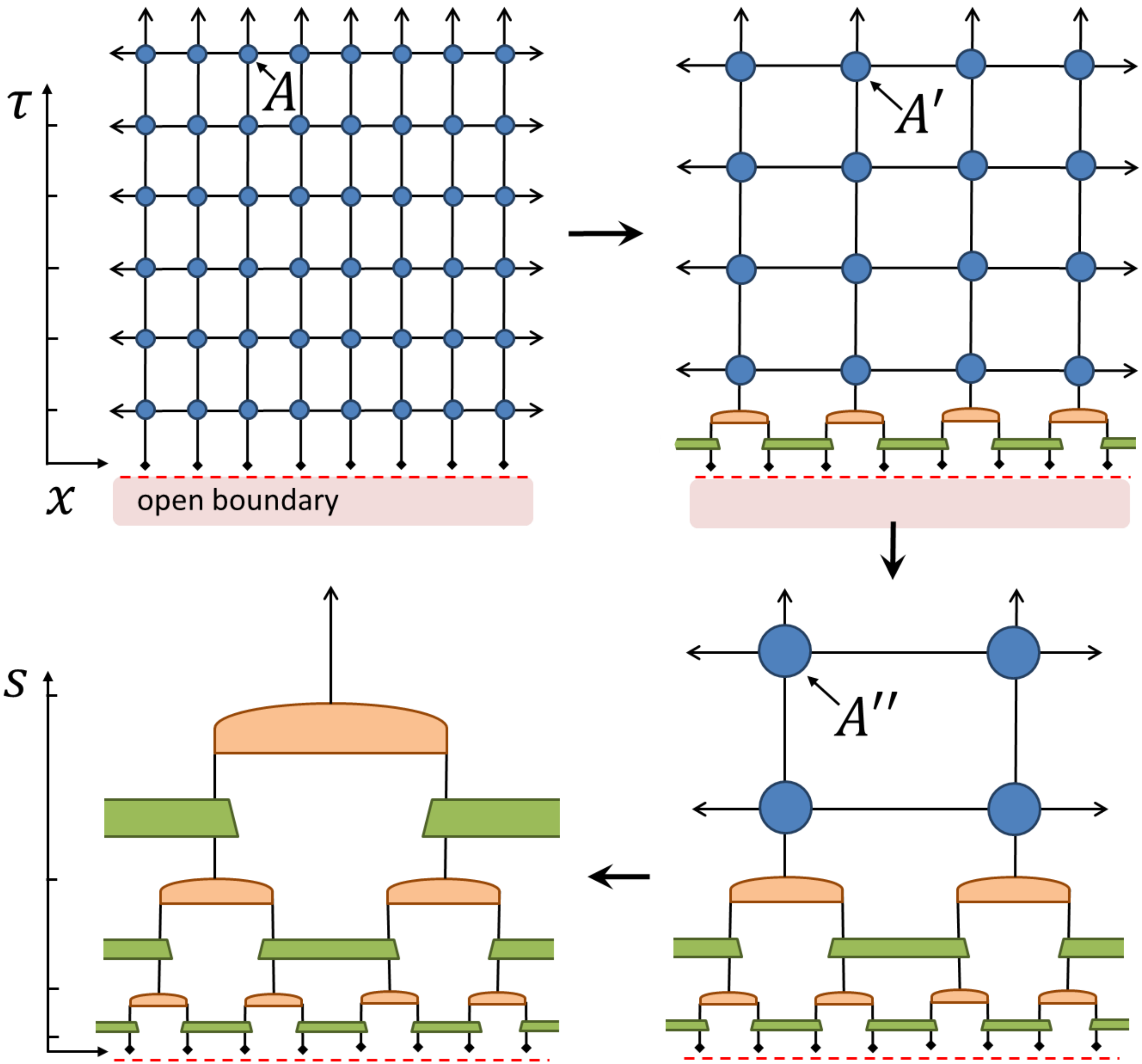
below critical
$T = 0.9\, T_c$
→ ordered (Z2) fixed point

critical
$T = T_c$
→ critical (scale-invariant) fixed point

above critical
$T = 1.1\, T_c$
→ disordered (trivial) fixed point

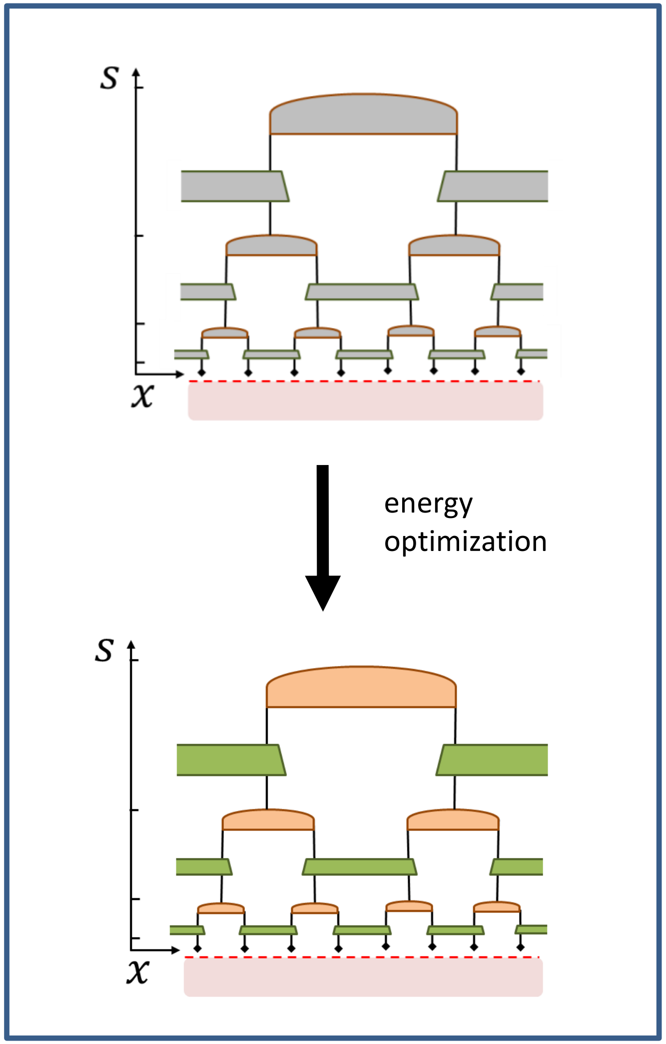
TNR yields MERA

Evenbly, Vidal, 2015



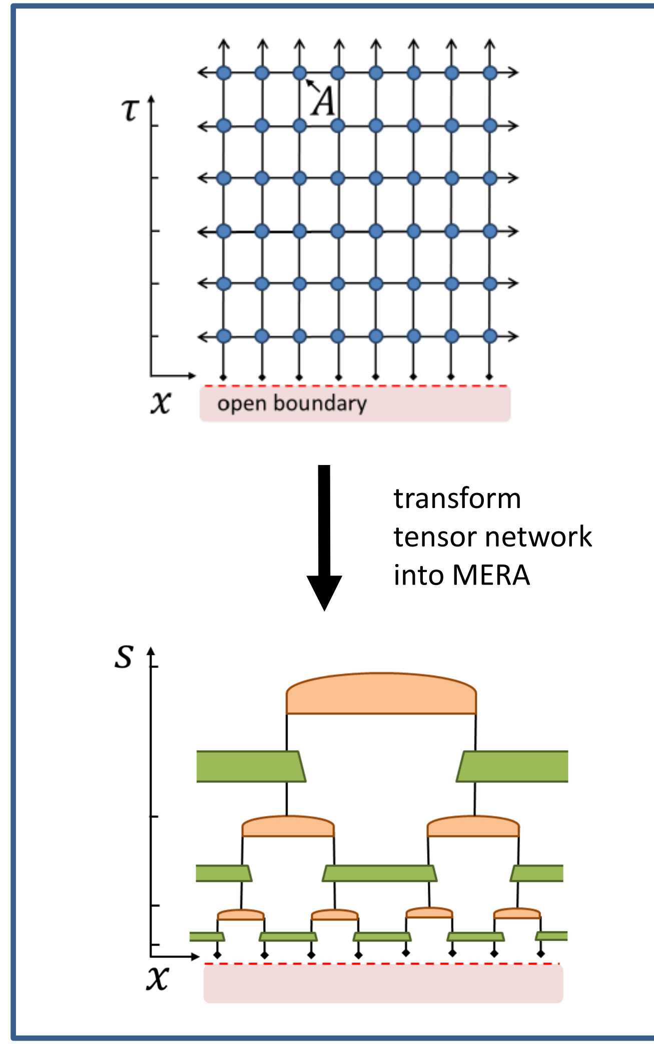
MERA = variational ansatz

MERA = by-product of TNR



energy minimization

- 1000s of iterations over scale
- local minima
- correct ground ?



TNR -> MERA

- single iteration over scale
- rewrite tensor network for ground state
- certificate of accuracy

Summary

- Reformulation of the RG
  using quantum information tools/concepts
  (quantum circuits, entanglement)

- Efficient representation of ground states (MERA)
  -> toy model for holography

- universal, non-perturbative, real-space RG approach

	Tensors	*Ground states*	*Hamiltonians*
3 RG flows	$A \rightarrow A' \rightarrow A'' \rightarrow \cdots$	$\vert\Psi\rangle \rightarrow \vert\Psi'\rangle \rightarrow \vert\Psi''\rangle \rightarrow \cdots$	$H \rightarrow H' \rightarrow H'' \rightarrow \cdots$

- Key ingredient: **removal of short-rage entanglement**

- Very accurate in 1+1 dimensions (Ising model, etc)

What about 2+1, 3+1? (and QCD?)

Entanglement renormalization
MERA

IQI, 2005

Sherman Fairchild Prize
Postdoctoral Fellow
(2003-2005)

Tensor network renormalization

IQIM, 2014

GLEN EVENBLY
Sherman Fairchild Prize
Postdoctoral Fellow
(2011-2014)

THANK YOU!