String Theory and the Real World

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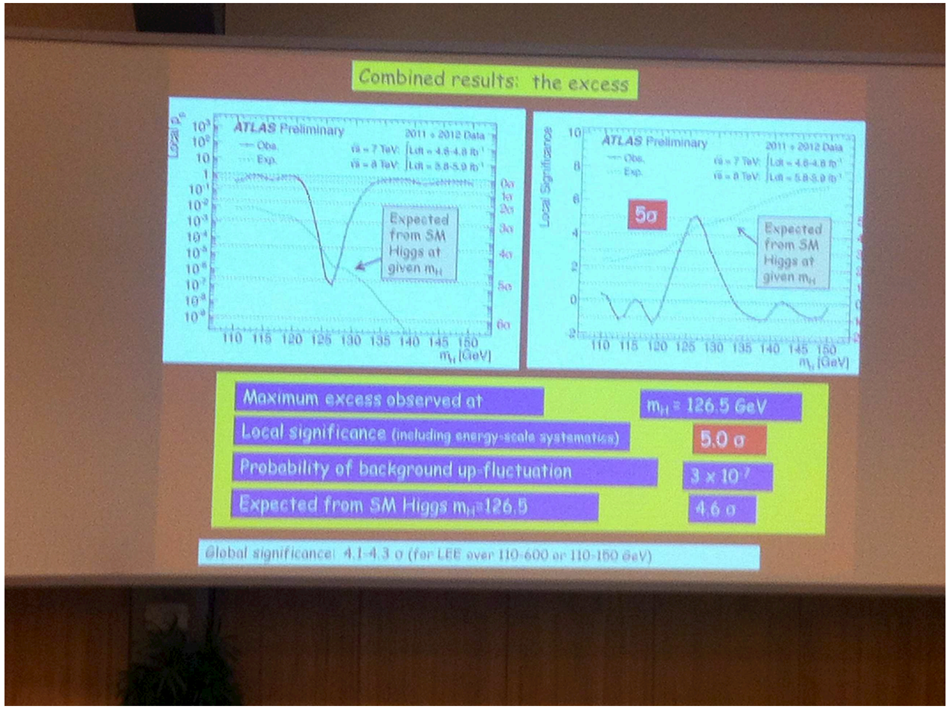
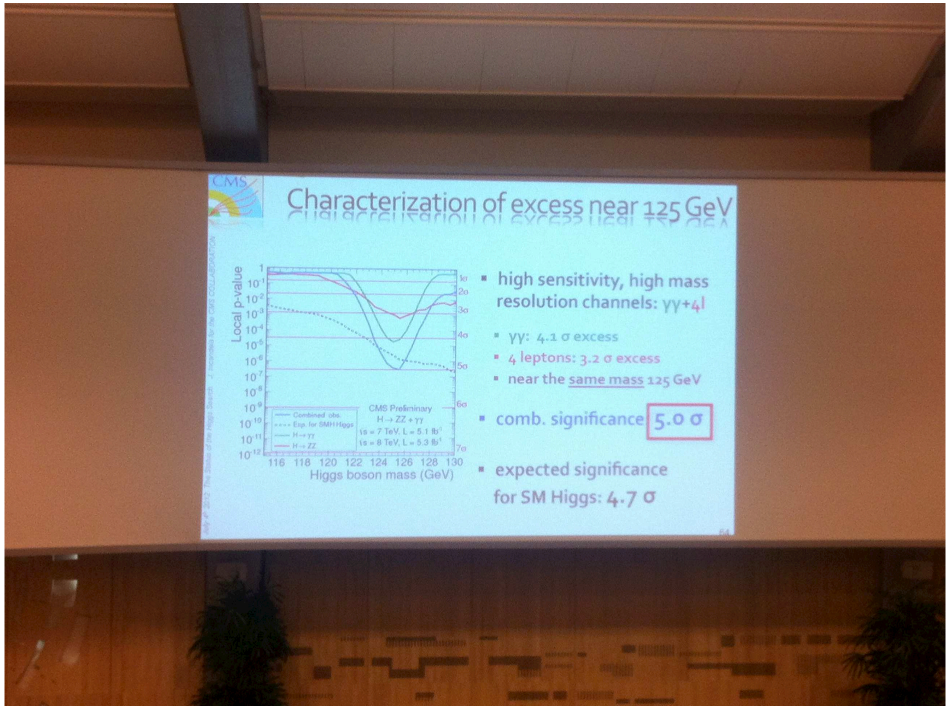
Some Real World Physics:
July 4, 2012, in the CERN foyer at 3am...

...and several hours later





What's next?

SM or not so standard?

Is String theory relevant for BSM?

Phenomenological Input

$SU(5)$ SUSY GUT:

- 3 generations of

$$\mathbf{10}_M = \begin{pmatrix} Q \sim (\mathbf{3},\mathbf{2})_{+1/6} \\ U^c \sim (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \\ E^c \sim (\mathbf{1},\mathbf{1})_{+1} \end{pmatrix}, \qquad \bar{\mathbf{5}}_M = \begin{pmatrix} D^c \sim (\bar{\mathbf{3}},\mathbf{1})_{+1/3} \\ L \sim (\mathbf{1},\mathbf{2})_{-1/2} \end{pmatrix}$$

- Higgses:

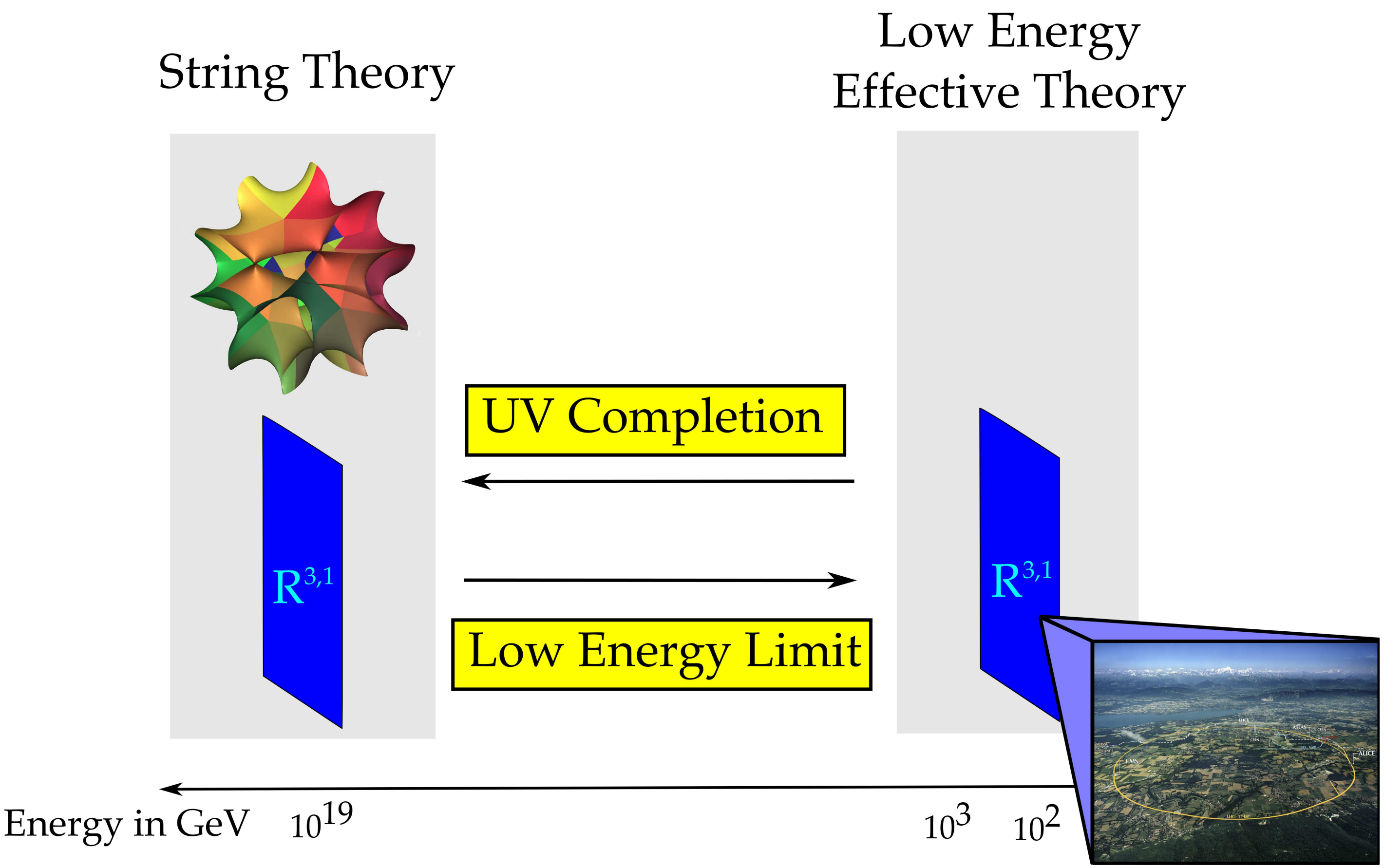
$$\mathbf{5}_H = \begin{pmatrix} H_u \sim (\mathbf{1},\mathbf{2})_{+1/2} \\ H_u^{(3)} \sim (\mathbf{3},\mathbf{1})_{-1/3} \end{pmatrix}, \qquad \bar{\mathbf{5}}_H = \begin{pmatrix} H_d \sim (\mathbf{1},\mathbf{2})_{-1/2} \\ H_d^{(3)} \sim (\bar{\mathbf{3}},\mathbf{1})_{+1/3} \end{pmatrix}$$

- $W \sim \lambda_t\, \mathbf{5}_H \times \mathbf{10}_M \times \mathbf{10}_M + \lambda_b\, \bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_M \times \mathbf{10}_M$
- GUT breaking, doublet-triplet splitting, avoiding proton decay operators
- SUSY-breaking, flavour, neutrino physics, etc.

Why is String Phenomenology hard?

Noble motivations:

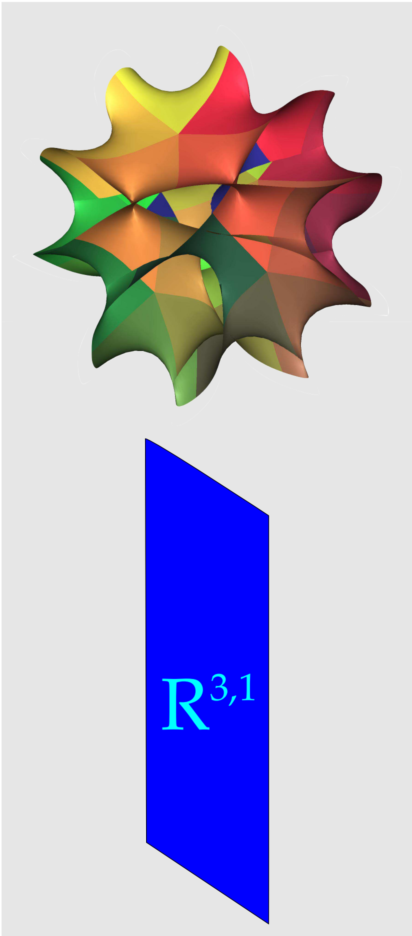
- Incorporates gravity and gauge theory
- UV completion
- SUSY, naturalness, quantum gravity, etc.



Standard Model: precise up to $O(10^2)$ GeV, but effective theory.

Which low energy theories have stringy UV completion? Are there any special characteristics of such vacua? $\Rightarrow$ Is it predictive?

String Theory and 4d Physics



- String theory unifies gauge theory and gravity

- Consistency requires extra dimensions
  $\Rightarrow$ Spacetime $\mathbb{R}^{3,1}$ emerges after
  compactification on manifold M

- Interface:

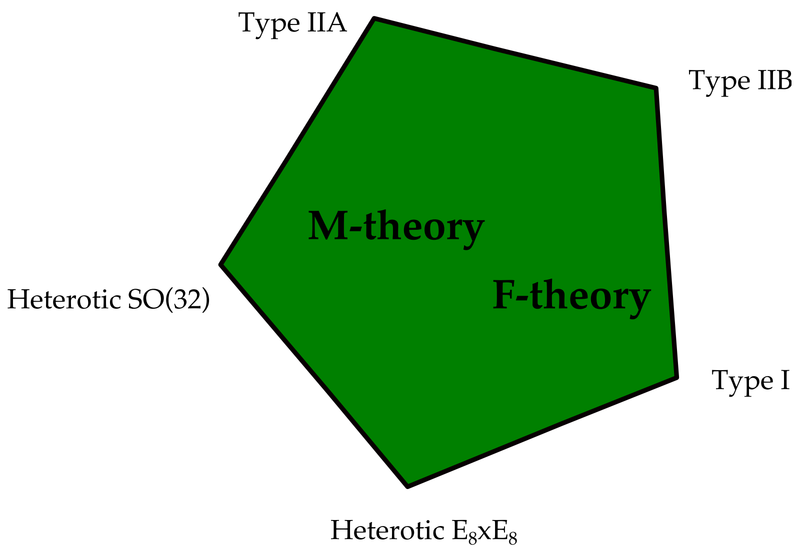
  Properties of 4d physics
  depend on the
  geometry of the compactification space

Examples:
geometric invariants, holonomy, singularities, symmetries of M encode gauge symmetries, supersymmetry, global symmetries of the low energy effective theory.

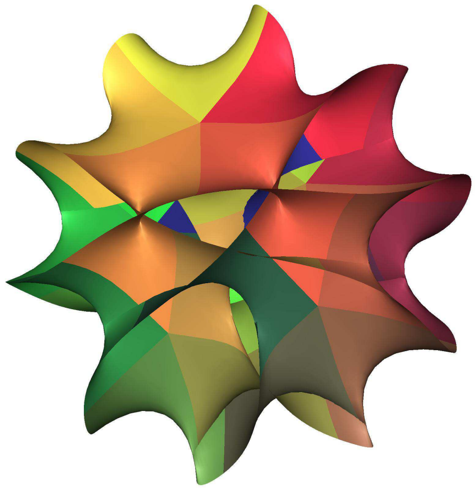
The Challenge

String theory vacua are not unique (disregarding even a dynamical selection mechanism)

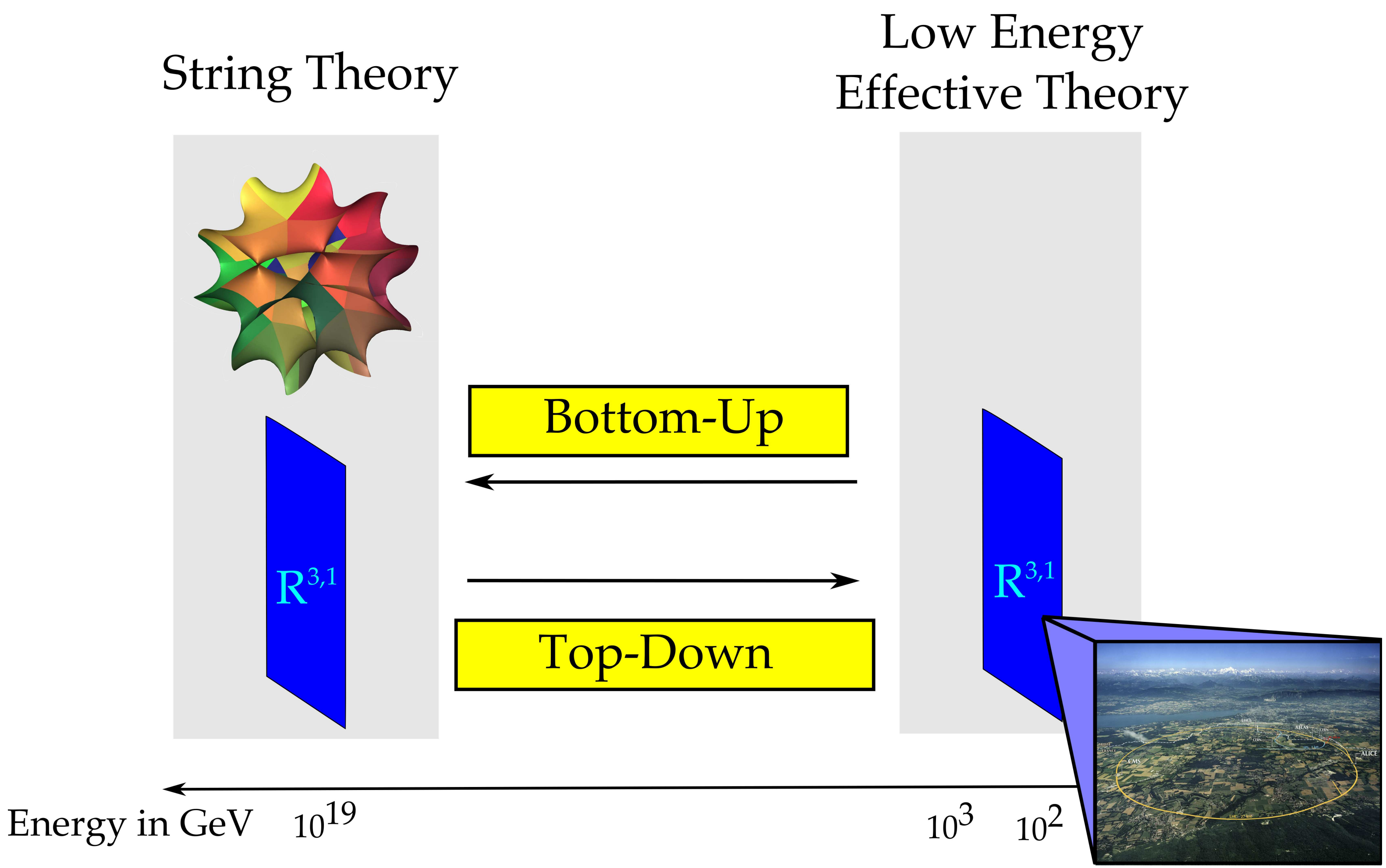
1. Choice of String Theory



2. Choice of Geometry



Top-Down versus Bottom-Up



Why String Phenomenology is hard:

- Bottom-up:
  Engineering semi-realistic models from ″local″ configurations
  # Advantage: realistic models
  # Disadvantage: no guarantee of global extension
  # Lacks constraining features (″too flexible″)

- “Top-down”:
  String theory T on space M yields 4d effective theory
  # Advantage: globally consistent
  # Disadvantage: Case-by-case: T on M' yields 4d effective theory′
  # Lacks characterization of general features (″too rigid″)

How to improve on this

Identify robust features of string compactifications, that are independent of the specific compactification geometries, and hold potentially even for the entire landscape of string vacua.

⇒ ”Stress-test approach”

Consistency with universal characteristics of the theory:

(1) Intermediate scale consistency: Higgs bundle

(2) Geometric consistency

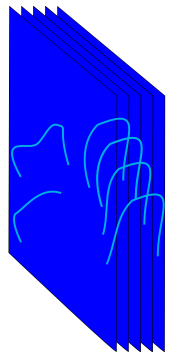
⇒ Such constraints can have imprints on the attainable 4d theories

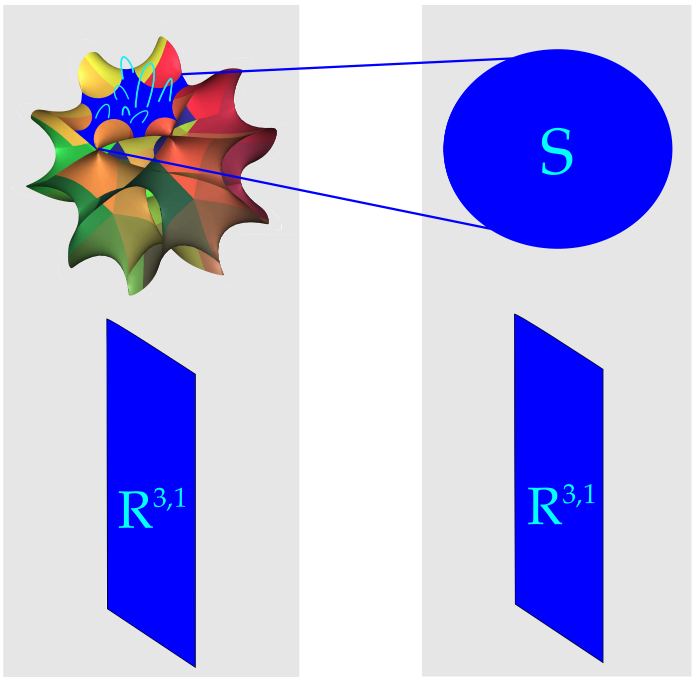
(1) Intermediate scale consistency

Low energy limit of open strings:

$$S_{SYM} \supset \frac{1}{g_{YM}^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} \,.$$

Endpoints sweep out "D-brane":





Compactification: D-branes wrap subspace

$$\mathbb{R}^{3,1} \times S \subset \mathbb{R}^{3,1} \times M$$

Open strings with energy E probe length scale $\ell \sim \frac{E}{T_{\text{string}}}$ away from brane

$\Rightarrow$ Low energy modes localize on $\mathbb{R}^{3,1} \times S$

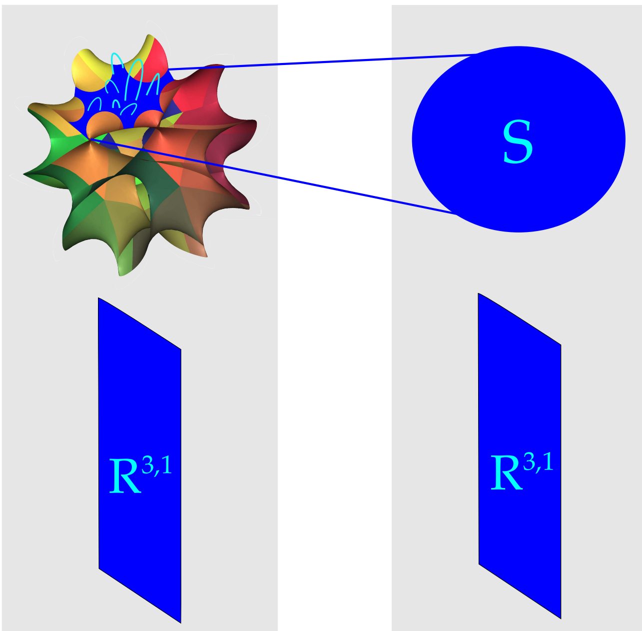
$\Rightarrow$ Gauge theory on $\mathbb{R}^{3,1} \times S$

Higgs Bundle and Hitchin Equations

Higgs bundle: Adjoint valued scalar field and connection (Φ, A) on S:
BPS equations "Hitchin Equations":

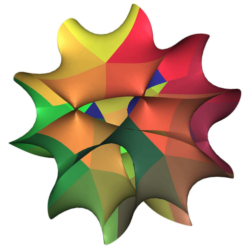
$$D_A\Phi = 0\,, \qquad F_A + [\Phi, \Phi^*] = 0$$

Higgs Bundle



- Universality:
  - IIB/F-theory: [Marsano, Saulina, SSN][10]
  - heterotic: [Candelas, etal]
  - M-theory on G_2: [Pantev, Wijnholt]
- Require: MSSM/GUT in 4d
- Geometric relevance:
  $\Rightarrow \langle\Phi\rangle$ encodes local geometry
  $\Rightarrow$ Seed to construct global geometries

(2) Geometric constraints



Lots of stuff to take care of:

- N=1 supersymmetry in d=4
  $\Rightarrow$ special holonomy of connection (Calabi-Yau, G_2)
- Moduli (volume of cycles, shapes). These are massless fields
  $\Rightarrow$ Moduli stabilization

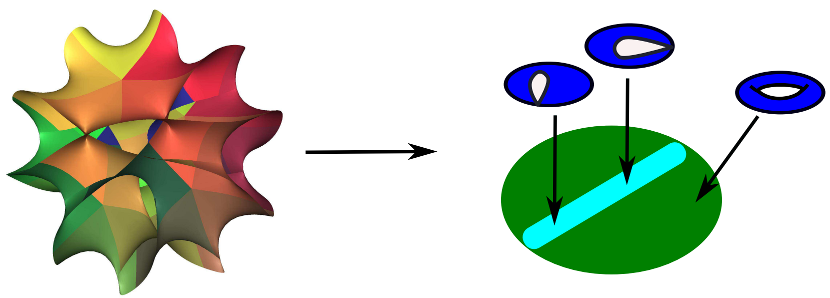
Identify robust features, which occur irrespective of concrete realization, in any compacticiation of this class.

Examples: Gauge dof's realized by singularities of geometry.

Stress-testing F-theory

Most active field in string phenomenology since 2008: Harvard, Caltech, UCSB/KITP, UPenn, KIPMU, MIT, London, Munich, Heidelberg...

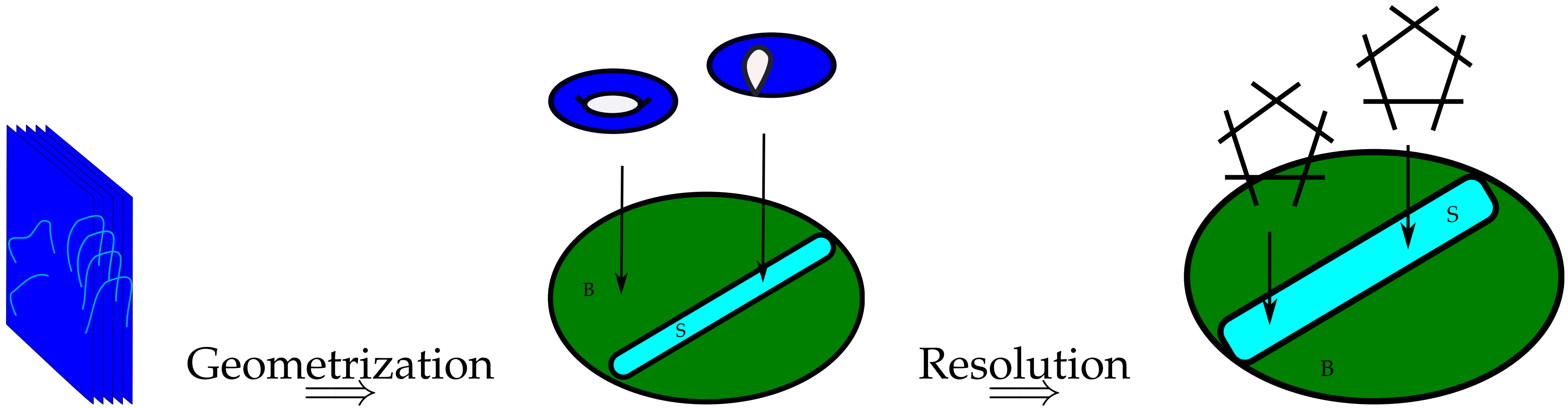
- F-theory [Morrison, Vafa]= non-perturbative Type IIB [Green, Schwarz]
- Geometries: Elliptically fibered Calabi-Yau



- Singularities of the elliptic fiber encode all the 4d physics:

Singularities of the Calabi-Yau $\Longleftrightarrow$ Gauge group, matter, Yukawas

Gauge theory from Singular Fibers



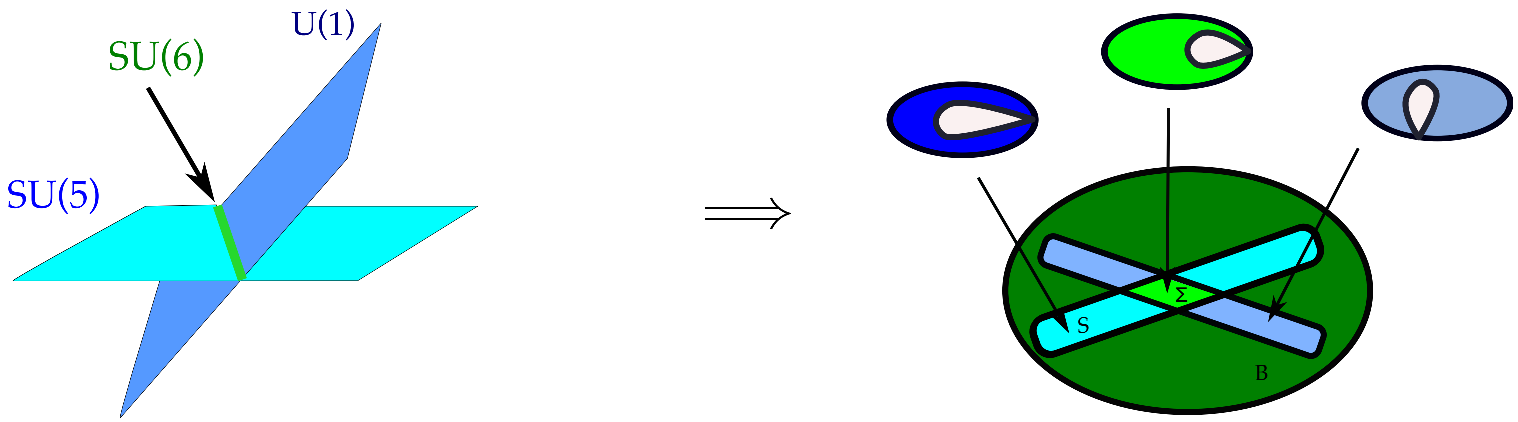
Geometrically:

Study resolution of singularities: smooth fibers are trees of $\mathbb{P}^1$s, intersecting in extended ADE Dynkin diagrams, e.g. SU(5)

Effective field theory:

$C_3 = A_i \wedge \omega_i^{(1,1)}$ and M2 wrapping modes give rise to gauge degrees of freedom.

Matter



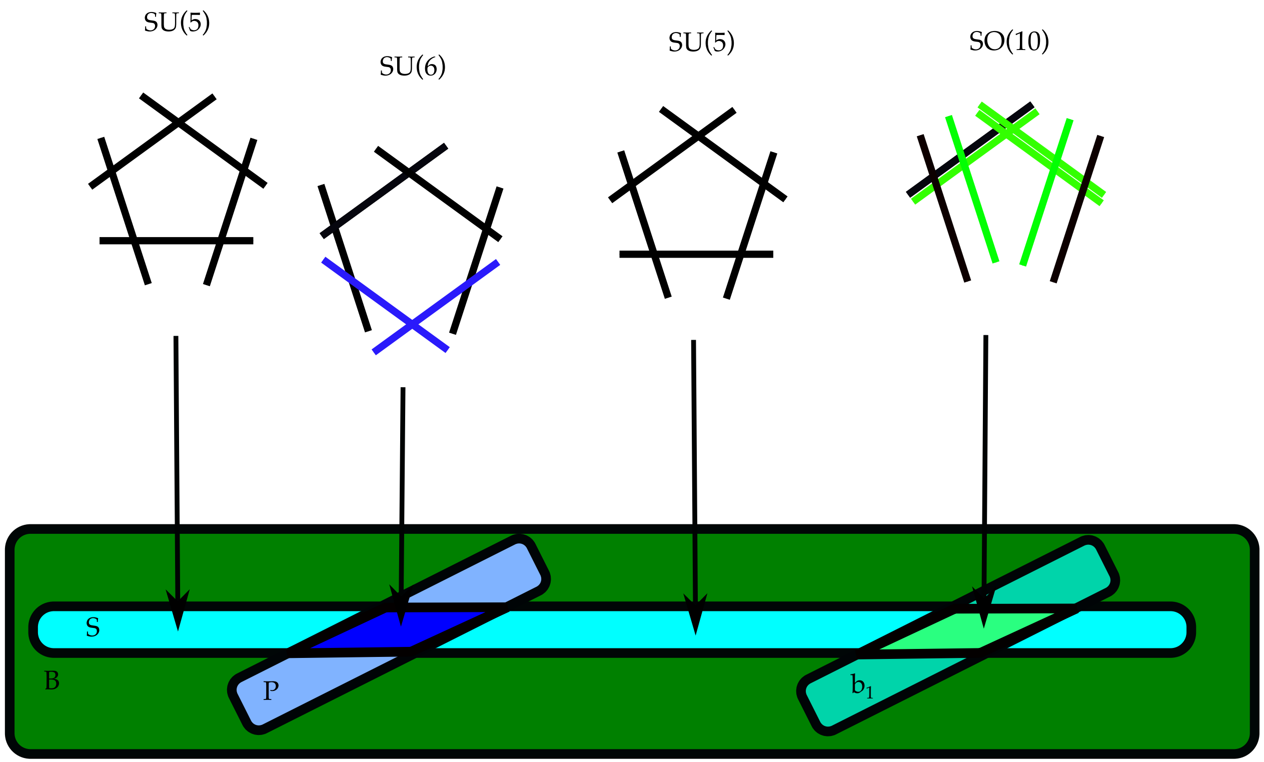
⇒ Matter is localized along codimension 2 loci Σ: Singularity worsens

⇒ Matter type determined by fiber type along codimension 2 locus:

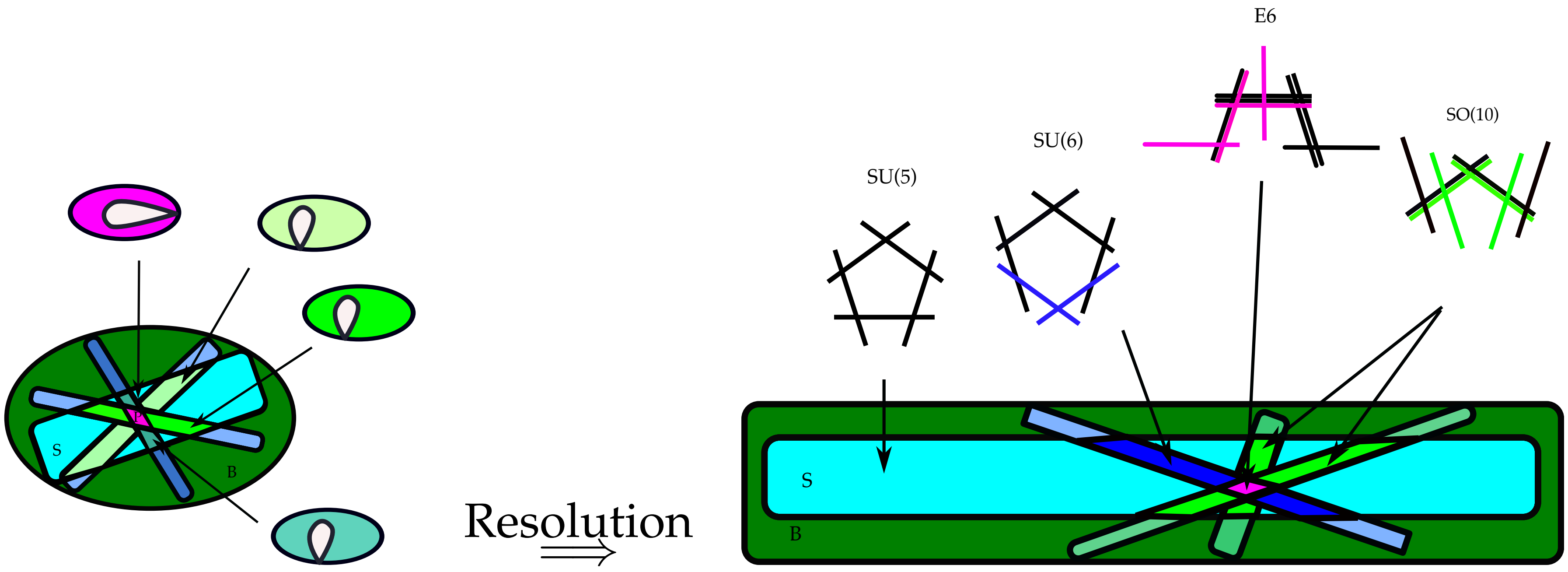
$$G_\Sigma = SO(10) \text{ or } SU(6) \quad \longrightarrow \quad SU(5) \times U(1)$$

gives **10** and **5** matter.

Possible matter determined by higher codimension structure of fibers. $\Rightarrow$ Classification of posssible codim 2 fibers?



Yukawa Couplings: Codimension 3



Classification of Singular Fibers

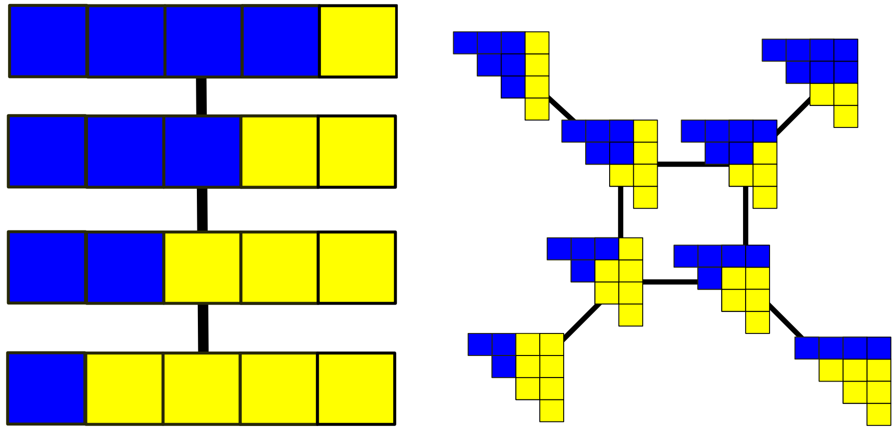
- Codim 1: Classic Algebraic Geometry [Kodaira-Néron]: Lie algebra $\mathfrak{g}$

Singular Fiber Codim 1	$\longleftrightarrow$	(Decorated) affine Dynkin diagram of $\mathfrak{g}$

- Codim 2: **R**= representation of $\mathfrak{g}$

Singular Fiber Codim 2	$\longleftrightarrow$	Box Graph = Decorated rep graph of **R**

E.g. **5** and **10** of $SU(5)$



[Hayashi, Lawrie, SSN][Hayashi, Lawrie, Morrison, SSN]

Why? Elliptic fibrations and Coulomb branches

[Hayashi, Lawrie, SSN], [Hayashi, Lawrie, Morrison, SSN]

- M-theory on resolved elliptically fibered Calabi-Yau
  $\Rightarrow$ Coulomb branch of 3d N=2 gauge theory:
  $\langle\phi\rangle \in \mathrm{CSA}(G) \Rightarrow G \to U(1)^{\mathrm{rank}(G)}$

$$\Rightarrow \text{Coulomb branch} \cong \mathbb{R}^{\mathrm{rank}(G)}/W_G = \text{Weyl chamber}$$

- Including matter, chiral multiplet Q, in representation $\mathbf{R}$, weight λ:

$$\mathcal{L} \supset |\phi\cdot\lambda|^2|Q|^2 \quad \Rightarrow \quad \text{walls in the Coulomb branch:} \quad \phi\cdot\lambda = 0$$

  Coulomb branch phases correspond to definite signs

$$\mathrm{sign}(\phi\cdot\lambda) = \epsilon = \pm 1 \qquad \forall\lambda \text{ in } \mathbf{R}$$

  i.e. sign-decorated representation graphs, "box graphs", of $\mathbf{R}$.

Coulomb Branch Phases: $\text{sign}(\phi \cdot \lambda) = \epsilon$

$\nearrow \swarrow$

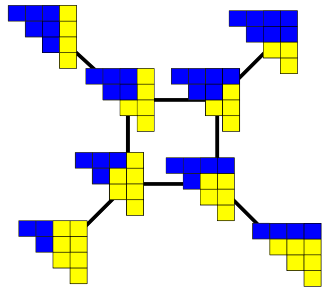
Box Graphs

$\searrow \nwarrow$

(Crepant) Resolutions of elliptic Calabi-Yaus

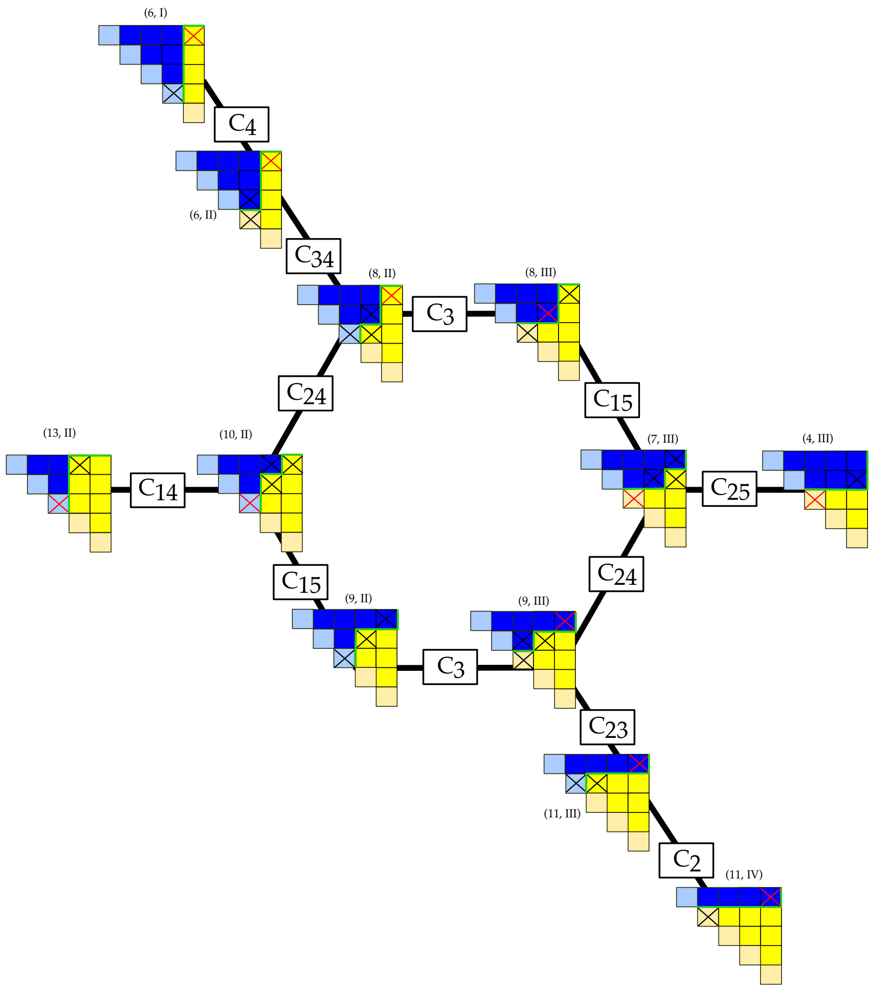
Fibers in codimension 2, which give rise to matter in representation $\mathbf{R}$ are characterized by decorated box graphs, based on representation graph:

$$\begin{aligned} \widetilde{\mathfrak{g}} \quad &\rightarrow \quad \mathfrak{g} \oplus \mathfrak{u}(1) \\ \text{Adj}(\widetilde{\mathfrak{g}}) \quad &\rightarrow \quad \text{Adj}(\mathfrak{g}) \oplus \text{Adj}(\mathfrak{u}(1)) \oplus \mathbf{R}_+ \oplus \overline{\mathbf{R}}_- \\ \text{Adj}(\mathfrak{so}(10)) \quad &\rightarrow \quad \text{Adj}(\mathfrak{su}(5)) \oplus \text{Adj}(\mathfrak{u}(1)) \oplus \mathbf{10}_+ \oplus \mathbf{10}_- \end{aligned}$$



Representation-theoretic characterization of flops: $W_{\widetilde{\mathfrak{g}}}/W_{\mathfrak{g}}$.

Fibers including Yukawa couplings for $SU(5)$ GUTs:



- Geometric repercussions: explicit construction of these smoothed singularities [Braun, SSN],[Esole, Shao,Yau], [Lawrie, SSN]

- Constrains all possible $U(1)$s comprehensively [Lawrie, SSN, Wong]

Matter spectra with $U(1)$	$\Longleftrightarrow$	Box graphs with rational sections

Mathematics:

Classication of higher codimension fibers with rational sections (Mordell-Weil group)

Physics:

- possible $U(1)$s in F-theory GUTs (without needing to construct the full set of Calabi-Yau)
- Next: implications on pheno

Phenomenology: towards the Real World

[Dolan, Marsano, SSN], [Krippendorf, SSN, Wong]

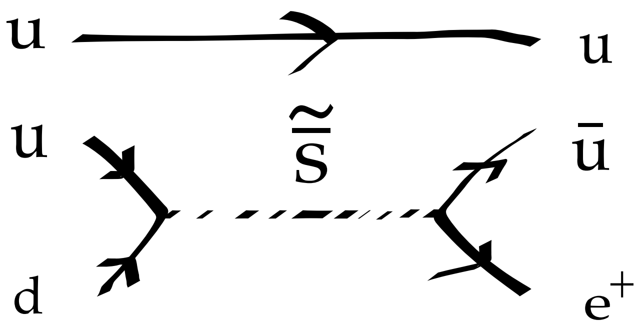
$U(1)$s for protecttion from Proton Decay: half-life $> 10^{36}$ years

Dimension 4:

Models generically contain B/L-violating operators (R-parity violating)

$$W \supset \lambda^0_{ijk} L_i L_j \bar{e}_k + \lambda^1_{ijk} \bar{d}_i L_j Q_k + \lambda^2_{ijk} \bar{d}_i \bar{d}_j \bar{u}_k$$

Proton decay rate $\sim \lambda^1_{11k} \lambda^2_{11k}$ leading to $p^+ \to \pi^0 + e^+$



Bound on coupling: $W \supset \lambda_{ijk} \overline{\mathbf{5}}_i \overline{\mathbf{5}}_j \mathbf{10}_k \,,$ $\lambda_{111} \leq \left(\frac{M_{SUSY}}{\text{TeV}} \right) 10^{-12}$

Fix: $U(1)_{B-L}$ or R-parity.

Dimension 5:

Coupling

$$\mathcal{L} \supset w_{ijkl} \mathbf{10}_i \mathbf{10}_j \mathbf{10}_k \overline{\mathbf{5}}_l$$

which gives rise to

$$w^1_{ijkl} Q_i Q_j Q_k L_l + w^2_{ijkl} \overline{u}_i \overline{u}_j \overline{e}_k \overline{d}_l + w^3_{ijkl} Q_i \overline{u}_j \overline{e}_k L_l$$

Bounds on couplings:

$$w_{112l} \leq 16\pi^2 \left(\frac{M_{SUSY}}{M^2_{GUT}} \right) \qquad l = 1, 2$$

Fix: $U(1)_{PQ}$

Characterize these by: absence of μ-term

$$q_{PQ}(H_u) + q_{PQ}(H_d) \neq 0$$

Reconstructing F-theory GUTs [Dolan, Marsano, SSN]

F-theory input: Comprehensive, universal characterization of $U(1)$s.
Pheno input: U(1)s suppress proton decay operators.
GUT breaking with $\langle F_Y \rangle \neq 0$ can induce mixed anomalies: MSSM-$U(1)$

Anomaly cancellation $\Rightarrow$ non-universal gaugino masses

$$M_1 : M_2 : M_3 = 1 : 2\alpha : 6\beta$$

Reconstrution via [Allanach, Lester, Parker, Webber] by studying channels

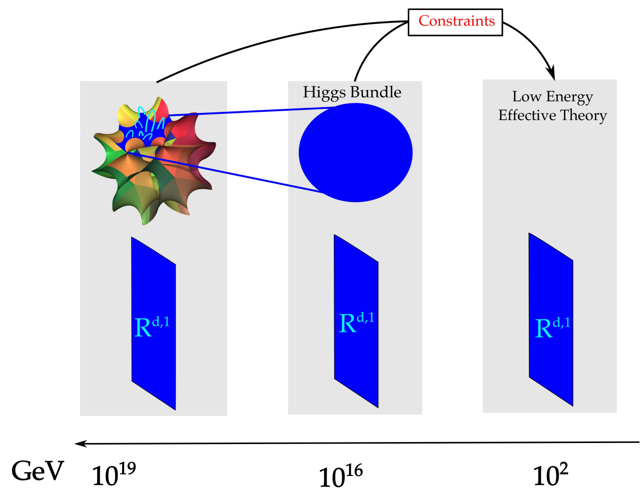
$$\tilde{q}_L \to \chi_2^0 q \to \tilde{l}_R^{\pm} l^{\mp} q \to \chi_1^0 l^{\pm} l^{\mp} q$$

Model	m_{ll}	m_{llq}^{edge}	m_{llq}^{thr}	m_{lq}^{high}	m_{lq}^{low}
mGMSB	138.7	1126	306	1102	396
F-theory GMSB	330.2	1011	550	856	688

$m_{ll} = (m^2_{\tilde{\chi}_0^1} - m^2_{\tilde{\ell}_R})(m^2_{\tilde{\ell}_R} - m^2_{\tilde{\chi}_0^2})/m^2_{\tilde{\ell}_R}$
= kinetic invariant for dilepton channel $\chi_2^0 \to \tilde{l}^{\pm} l^{\mp} \to \chi_1^0 l^{\pm} l^{\mp}$

Summary and Onwards

- Universality of Higgs bundle description
  Generalization: develop in M-theory on G_2, find inter-string-universalities
- Universality of the singularity structure in F-theory
  Generalization: extend this to other corners of the string theory landscape.
- Extract robust features of the geometries.
  Main mathematical challenge: M-theory on G_2 manifolds.



Thank you