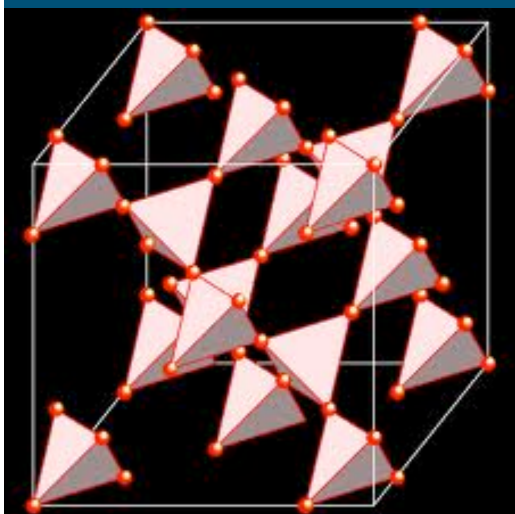
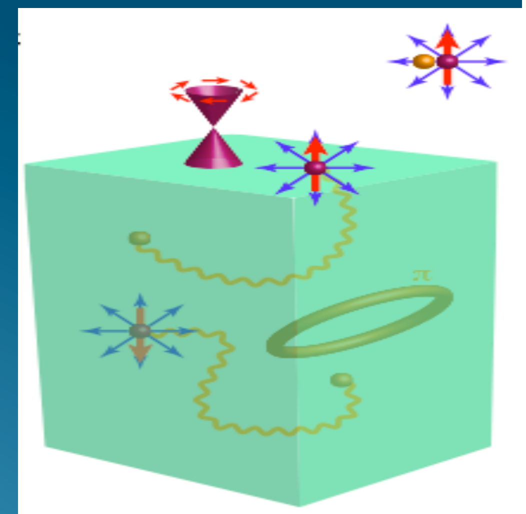
Fractionalized Topological Insulators



Gregory A. Fiete
University of Texas at Austin



Collaborators

Mehdi Kargarian

Victor Chua

Joseph Maciejko

Related Works:

M. Kargarian and GAF, *Phys. Rev. Lett*. (2013)

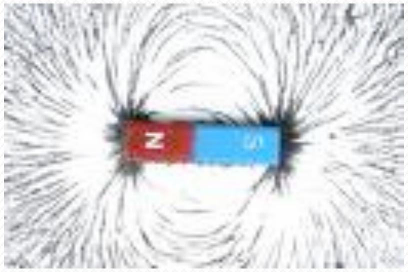
J. Maciejko, V. Chua, and GAF *Phys. Rev. Lett*. (2014)

Outline

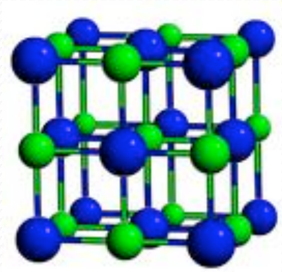
- A brief history of “exotic” phenomena in condensed matter physics
- Topological insulators
- Fractionalized topological insulators
- Wrap up

A major focus in condensed matter physics: "Phases" and "Orders"

- Typically large number of particles > 10^{18}, so close to thermodynamic limit.
- "Classification" of phases by order is a central research topic.
- Local (order parameter) vs. Global (topological) order



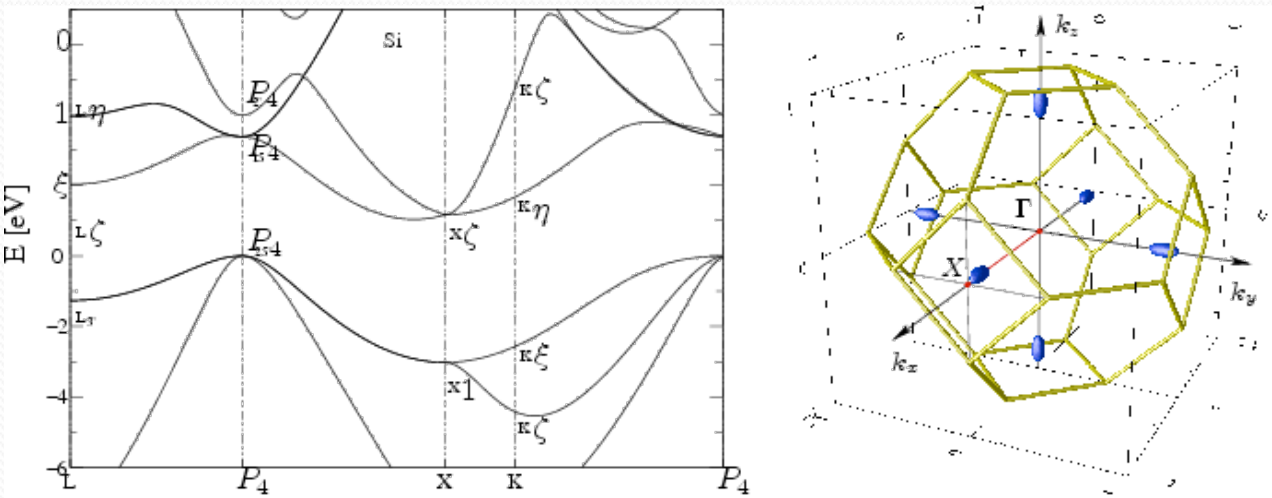
Magnetic



Crystalline

The "Good Old Days"

- Electronic band structure for crystalline solids

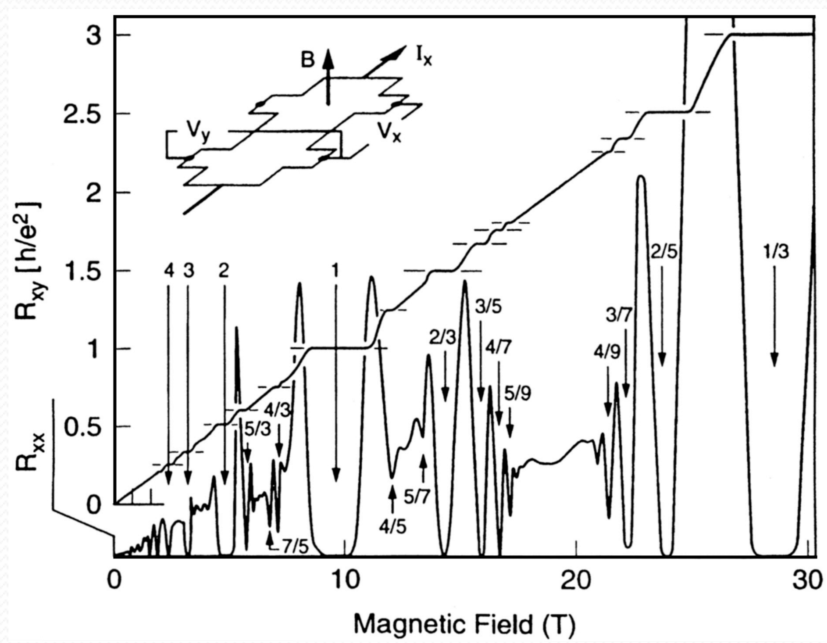


(a) Band diagram of silicon. (b) First conduction band valleys.

- Underpins most modern electronics and communication devices, e.g., transistor.
- Not Generic! Depends on fundamental excitations being electron-like.

Fractional quantum Hall effect: Non-local topological order

- Discovery: early 1980s
- No broken spatial symmetries
- Fractional charges; interacting
- Bulk-boundary correspondence
- Chern-Simons topological field theory:

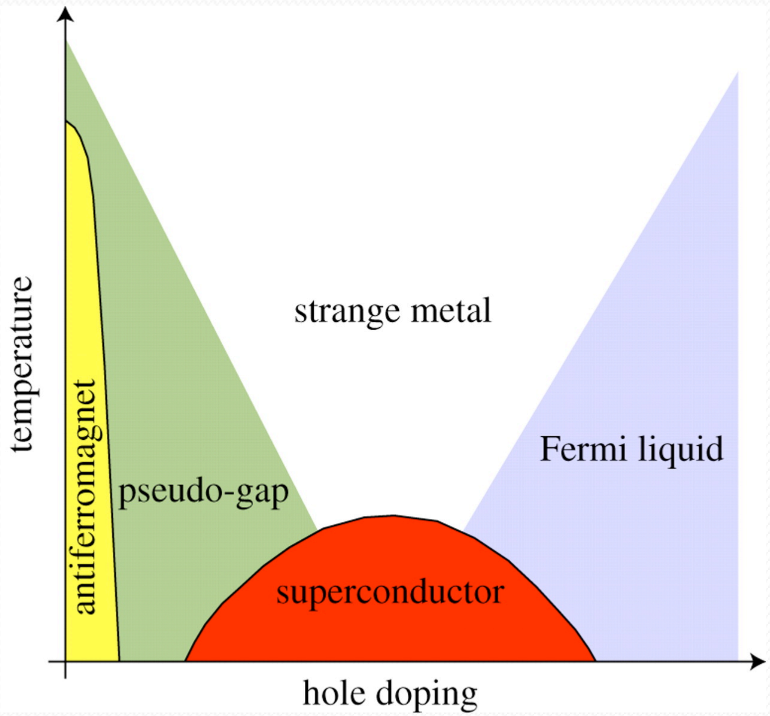


$$\mathcal{L}_{CS} = -m\frac{1}{4\pi}a_\nu\partial_\nu a_\lambda\epsilon^{\mu\nu\lambda} + \frac{e}{2\pi}A_\mu\partial_\nu a_\lambda\epsilon^{\mu\nu\lambda}$$

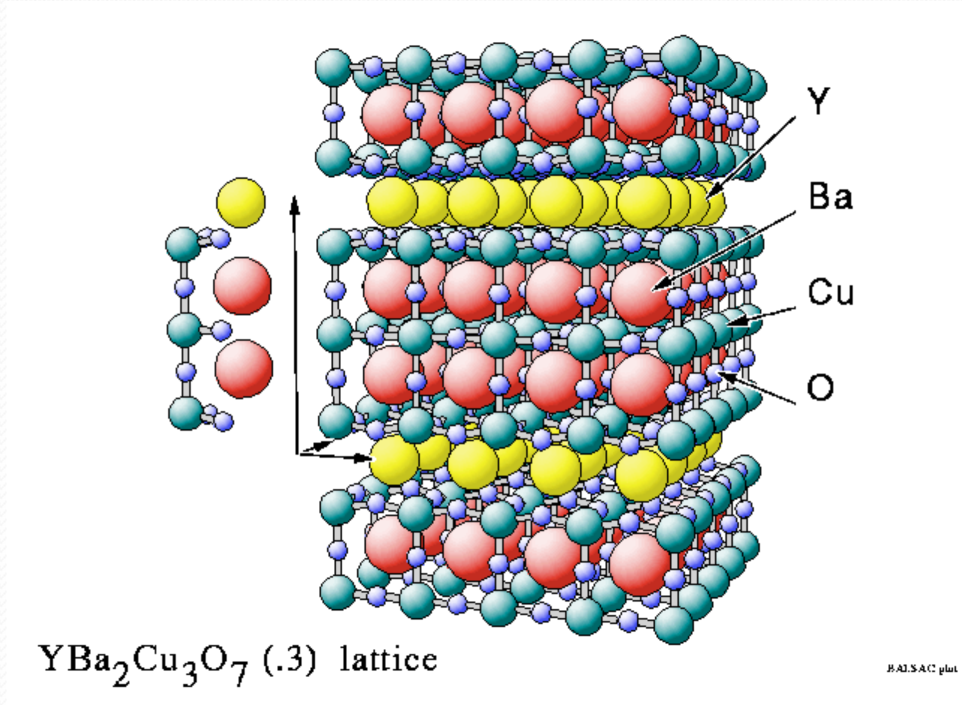
$$+la_\mu j^\mu + \text{kinetic/potential energy}$$

High temperature superconductivity: Correlated physics in transition metal oxides

Generic phase diagram



Example parent compound



Discovery: Mid-1980s

Highest transition temperature is approaching 200 Kelvin.

Influence of FQHE and high-Tc superconductivity

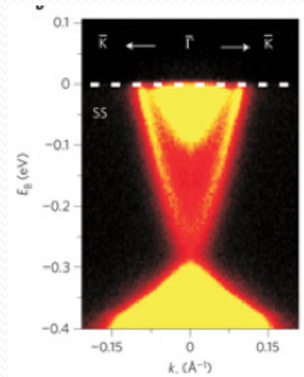
- FQHE gave a concrete example of a class of fractionalized electronic states and non-local topological order: Inspired many subsequent developments, including “fractionalization” ideas for high-temperature superconductors and top insulators.

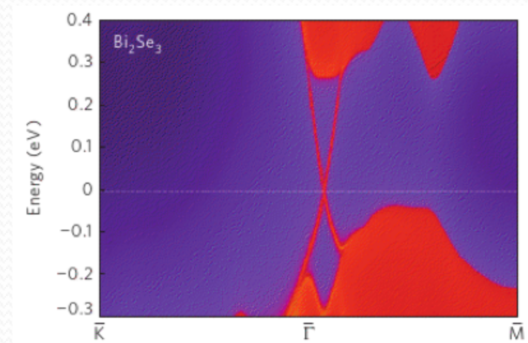
- High temperature SC with its unusual insulating, correlated parent state inspired many novel ideas in quantum magnetism, notably “quantum spin liquids”, and quantum criticality more generally. Also drove materials science developments in oxides.

New kid on the block-Topological Insulators (3-d): Bi_2Se_3, Bi_2Te_3,$TlBiSe_2$, Bi_2Te_2Se (ARPES identified)

- Bi_2Se_3

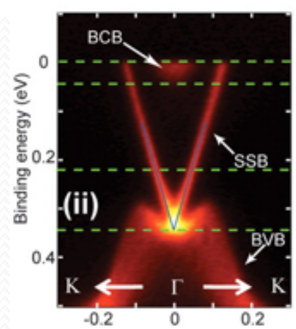
  Xia *et al*. Nat. Phys. **5**, 398 (2009)

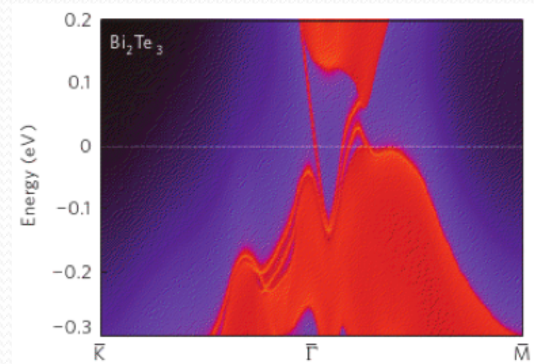




- Bi_2Te_3

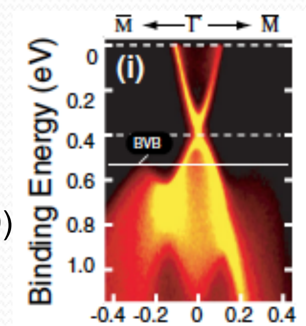
  Chen *et al*. Science **325**, 178 (2009)





- $TlBiSe_2$

  Sato *et al*. PRL **105**, 136802 (2010)
  Kuroda *et al*. PRL **105**, 146801 (2010)

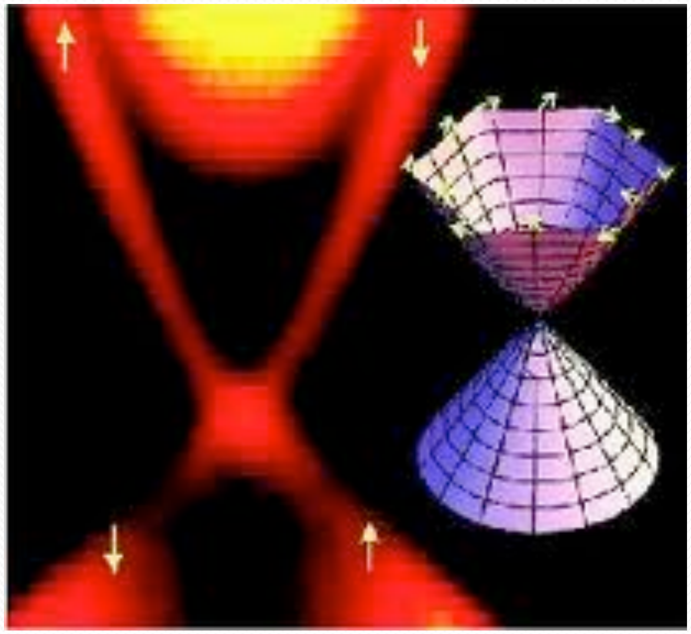


DFT: Zhang *et al.* Nat. Phys. **5**, 438 (2009)

- Bi_2Te_2Se: Single Dirac cone & most insulating bulk

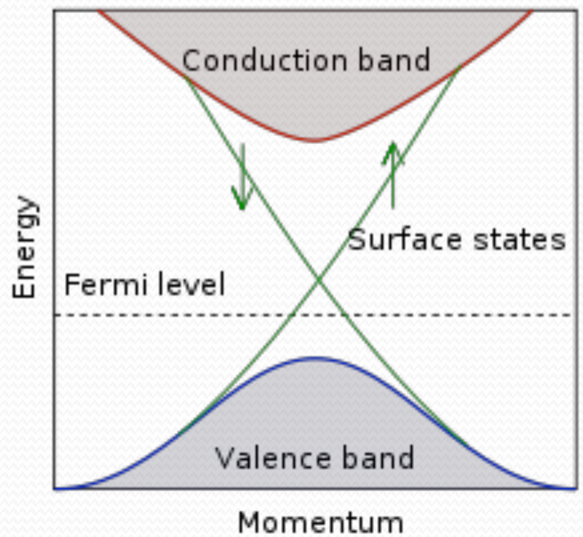
  Z. Ren *et al.* PRB **82**, 241306 (2010) (>70% surface conductivity)

Simple Picture of Topological Insulator Surface States: Spin-orbit coupling driven, spin-momentum locking



Surface state band structure

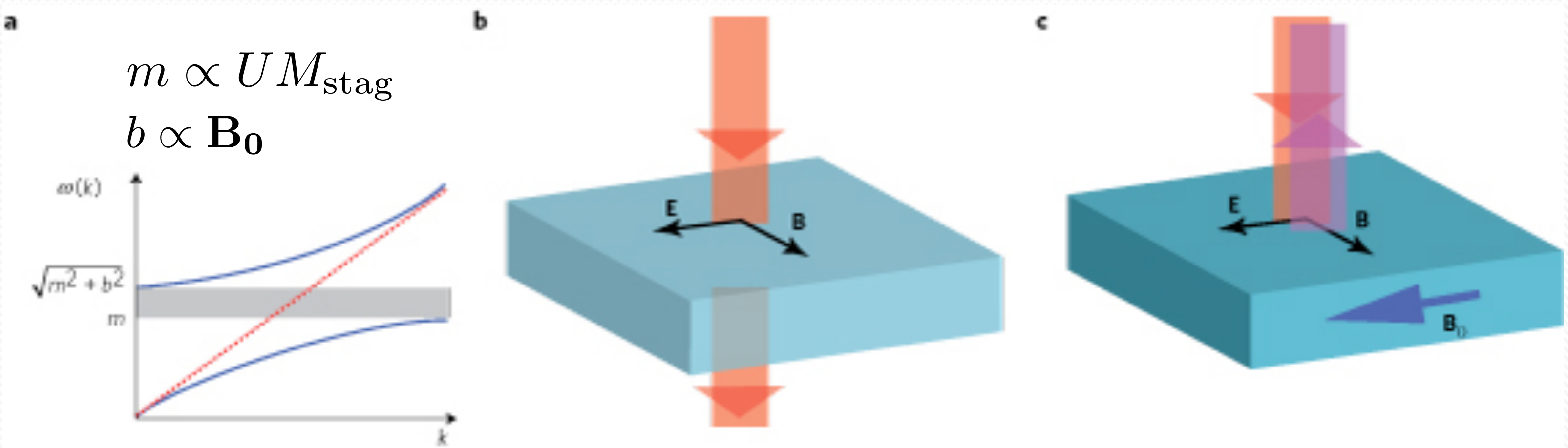
Topological Insulators



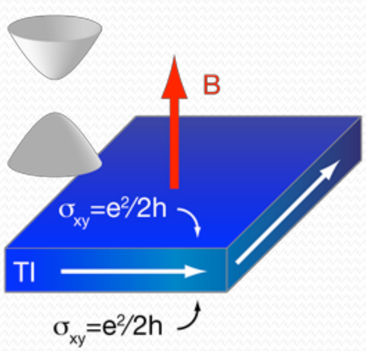
Theory: Fu and Kane, PRL (2007); Moore and Balents, PRB (2007); Roy, PRB (2009)
Experiment: Hsieh,..Hasan, Nature (2008) [$Bi_{1-x}Sb_x$]

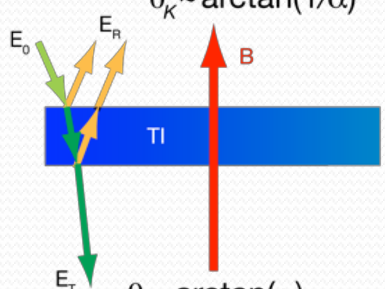
What can you do with a topological insulator?

Optical modulator proposal: Li, Wang, Qi, Zhang, Nat. Phys. (2010)



$$\sqrt{m^2+b^2} - m \approx 0.1\mathrm{meV}$$ (Far infra-red)





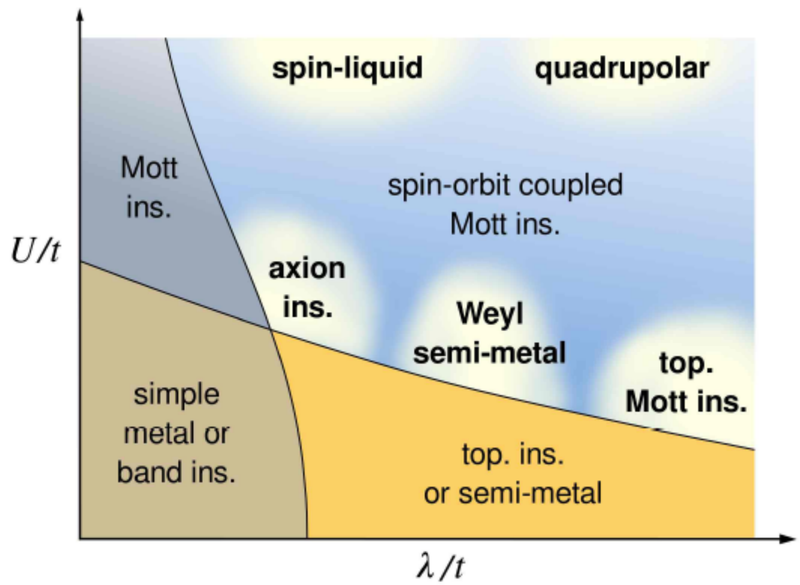
Quantized Faraday and giant Kerr rotation:
Tse and MacDonald, PRL (2010)

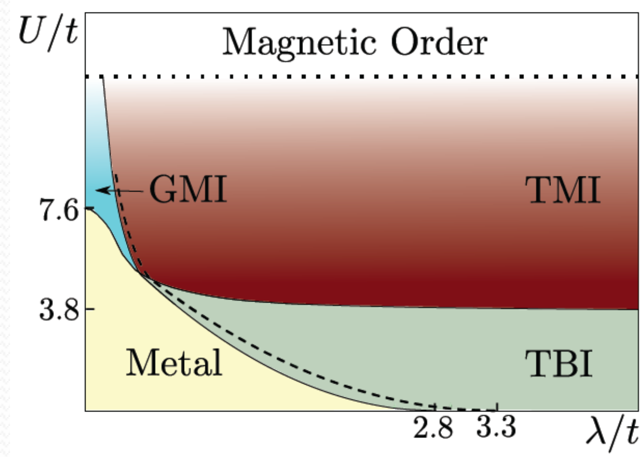
$$\theta_F \approx \alpha \qquad \theta_K \approx \frac{\pi}{2}$$

Moving toward fractionalized topological insulators

- The "unusual" regime of strong spin-orbit coupling and significant electron-electron interactions is particularly interesting.

W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents (2014)



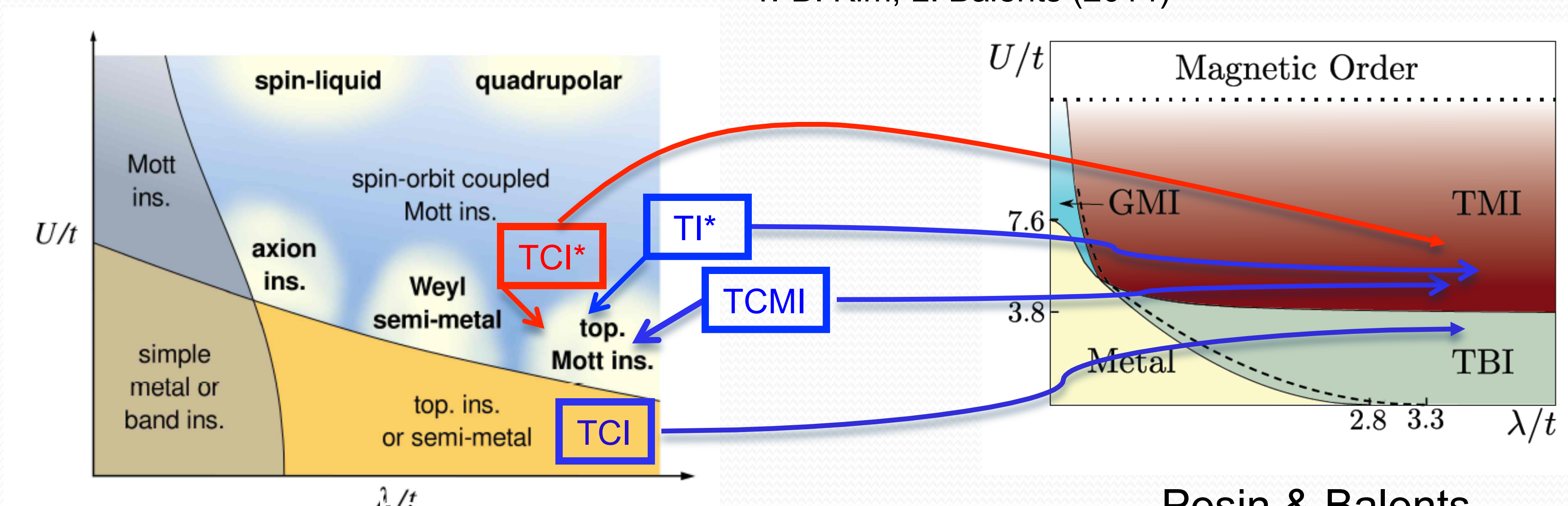


Pesin & Balents Nat. Phys. 2010

Decorating the Pesin-Balents Phase Diagram with New Topological Phases

- The "unusual" regime of strong spin-orbit coupling and significant electron-electron interactions is particularly interesting.

W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents (2014)



Pesin & Balents Nat. Phys. 2010

Topological Crystalline Insulators: Mirror Chern Number

- Predicted by Liang Fu and collaborators:

  --T. H. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, L. Fu, Nat Commun, 3, 982 (2012). SnTe Material class

  --J. Liu, W. Duan, L. Fu PRB (2013).

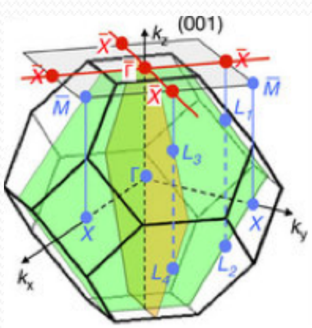
  IV-VI semiconductors

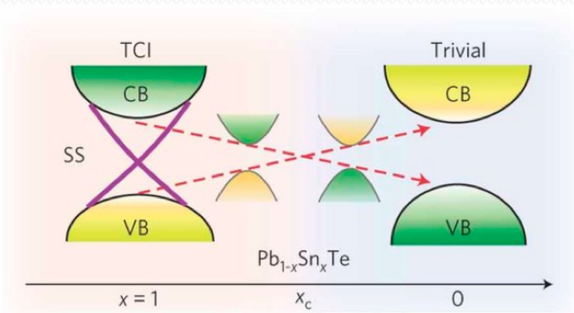
- Realized essentially simultaneously by three groups:

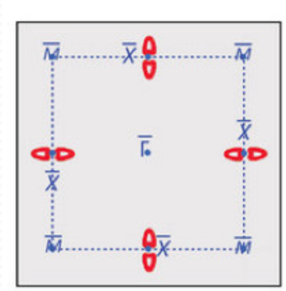
  S.-Y. Xu, … M. Z. Hasan, Nat Commun, 3, 1192 (2012) $Pb_{1-x}Sn_xTe$

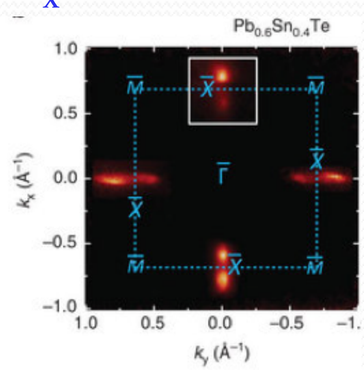
  Y. Tanaka, … Y. Ando, Nat Phys, 8, 800 (2012). SnTe

  P. Dziawa, … T. Story, Nat Mater, 11, 1023 (2012). $Pb_{1-x}Sn_xSe$



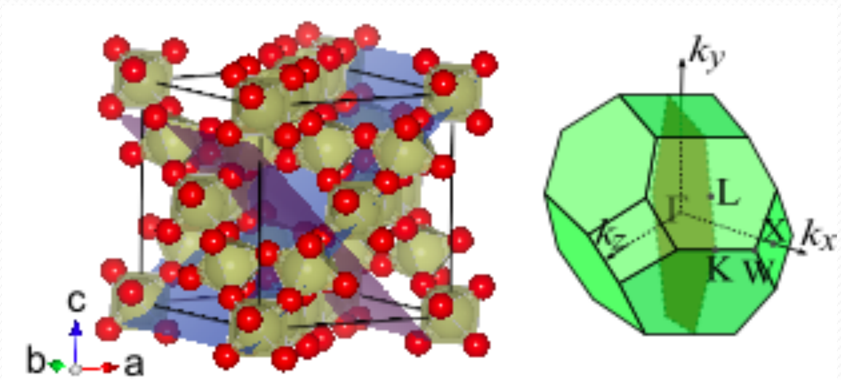






Topological Crystalline Insulator (TCI) in $A_2Ir_2O_7$

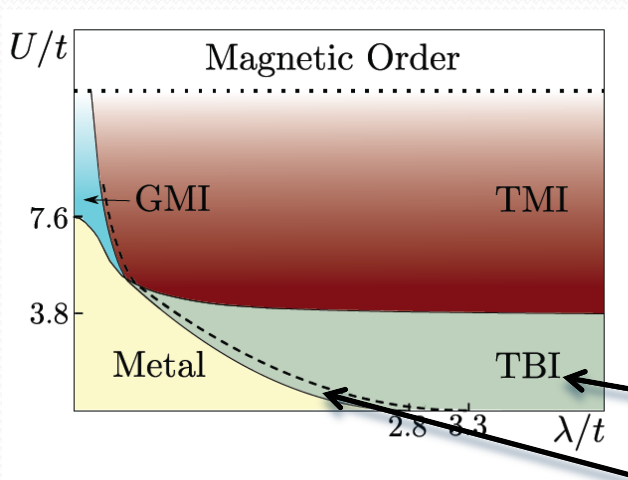
M. Kargarian and GAF PRL (2013)



$$H_0 = \sum_i t_i^{\gamma\gamma'} d_{i\gamma}^\dagger d_{i\gamma'} + \sum_{<ij>} (T_{o,ij}^{\gamma\gamma'} + T_{d,ij}^{\gamma\gamma'}) d_{i\gamma}^\dagger d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$

4-2-2



Pesin and Balents (2012)

States on the mirror plane can be organized by mirror eigenvalue +/- i

Mirror Chern number is difference of Chern numbers for each mirror eigenstate:

$$n_M = (n_{+i} - n_{-i})/2$$

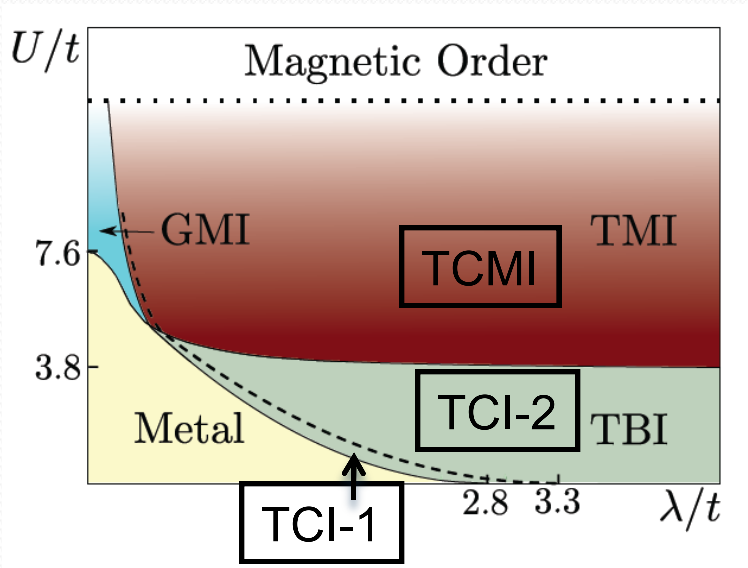
$$n_M = -1 \text{ for } \lambda > \lambda_c$$

$$n_M = +1 \text{ for } \lambda < \lambda_c$$

Teo, Fu, Kane PRB (2007)

"TCMI" from slave rotors: Refined Mean-field Phase Diagram

M. Kargarian and GAF PRL (2013)



Pesin and Balents Nat. Phys. (2010)

$$H_0 = \sum_i t_i^{\gamma\gamma'} d^\dagger_{i\gamma} d_{i\gamma'} + \sum_{<ij>} (T^{\gamma\gamma'}_{o,ij} + T^{\gamma\gamma'}_{d,ij}) d^\dagger_{i\gamma} d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$

4-2-2

$$H_U = U \sum_i \left(\sum_\gamma n_{i\gamma} - n_d\right)^2$$

$$d_{j\gamma} = e^{i\theta_j} f_{j\gamma}$$

charge

spin

Find that TMI is also TCMI—a spin liquid with topological band structure protected by both time-reversal and mirror symmetries. TBI is two "flavors" of TCI.

A Correlated Cousin for the TMI (and TCMI) Field Theory for TI* (and TCI*): General

J. Maciejko, V. Chua, GAF PRL (2014)

$$H = \sum_{rr'} \sum_{\alpha\beta} t^{rr'}_{\alpha\beta} c^\dagger_{r\alpha} c_{r'\beta} + \frac{U}{2} \sum_r \left(\sum_{\alpha=\uparrow,\downarrow} n_{r\alpha} - 1 \right)^2$$

Topological U=0

psuedo-spin

$$c_{r\alpha} = f_{r\alpha} \tau^x_r$$

Local constraint: G_r=1 $\quad G_r = (-1)^{\sum_\alpha f^\dagger_{r\alpha} f_{r\alpha} + \frac{1}{2}(\tau^z_r - 1)}$

Compute: $Z = \mathrm{Tr}(\mathrm{e}^{-\beta \mathrm{H}} \mathrm{P})$ where $P = \prod_r [(1 + G_r)/2]$

Find:

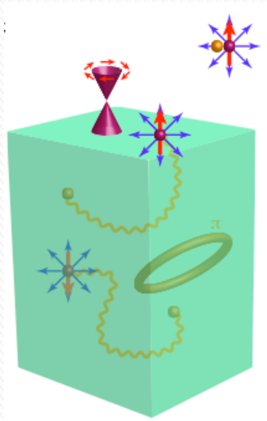
$$Z = \int D\bar{f}_{i\alpha} D f_{i\alpha} \sum_{\{\tau^x_i\}} \sum_{\{\sigma_{ij}\}} e^{-S_{\mathbb{Z}_2}[\bar{f}, f, \tau^x, \sigma]}$$

$$S_{\mathbb{Z}_2} = S_{\tau^x} + S_f + S_B$$

$$S_{\tau^x} = -\kappa \sum_{ij} \tau^x_i \sigma_{ij} \tau^x_j,$$

$$S_f = -\sum_{ij} \sum_{\alpha\beta} t^{ij}_{\alpha\beta} \bar{f}_{i\alpha} \sigma_{ij} f_{j\beta},$$

$$e^{-S_B} = \prod_{i,j=i-\hat{\tau}} \sigma_{ij}$$



Field Theory for TI*: Low-energy

J. Maciejko, V. Chua, GAF PRL (2014)

Write Z_2 gauge theory in terms of U(1) gauge theory:

$$\sigma_{ij} = e^{ia_{ij}}$$

Ukawa, Windey, Guth PRB (1980)

$$S_{U(1)} = S_{\mathbb{Z}_2}[\sigma_{ij} = e^{ia_{ij}}] + S_n \quad \text{where} \quad S_n = -ip\sum_{ij} n_{ij}a_{ij} \qquad \text{p=2}$$

Integrate out gapped slave fermions and focus on ***deconfined*** TI* phase: U(1) gauge field weakly coupled, so lattice unimportant -> take the continuum limit. $n_{ij} \to n_\mu, a_{ij} \to a_\mu$ U(1) gauge invariance requires $\partial_\mu n_\mu = 0 \to n_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho}$ where $b_{\mu\nu}$ is a compact U(1) 2-form.

$$S_n \to \frac{p}{4\pi}\int d^4x\, \epsilon^{\mu\nu\lambda\rho} b_{\mu\nu}\partial_\lambda a_\rho$$

(3+1)-d level p *BF* term

Cho & Moore Ann. Phys. (2011)
Chan, Hughes, Ryu, Fradkin PRB (2013)

After integrating out slave fermions, $\mathcal{L}_{\mathrm{TI}^*} = \frac{p}{4\pi}\epsilon^{\mu\nu\lambda\rho}b_{\mu\nu}\partial_\lambda(a_\rho - eA_\rho) + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\lambda\rho}f_{\mu\nu}f_{\lambda\rho}$

Integrate out $b_{\mu\nu}$: $\mathcal{L}_{\mathrm{em}} = \frac{\theta e^2}{32\pi^2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$

Topological degeneracy=8 on T^3
Non-trivial braid angle of $2\pi/p$.

TI-like magneto-electric response, but non-trivial ground state degeneracy and braiding statistics

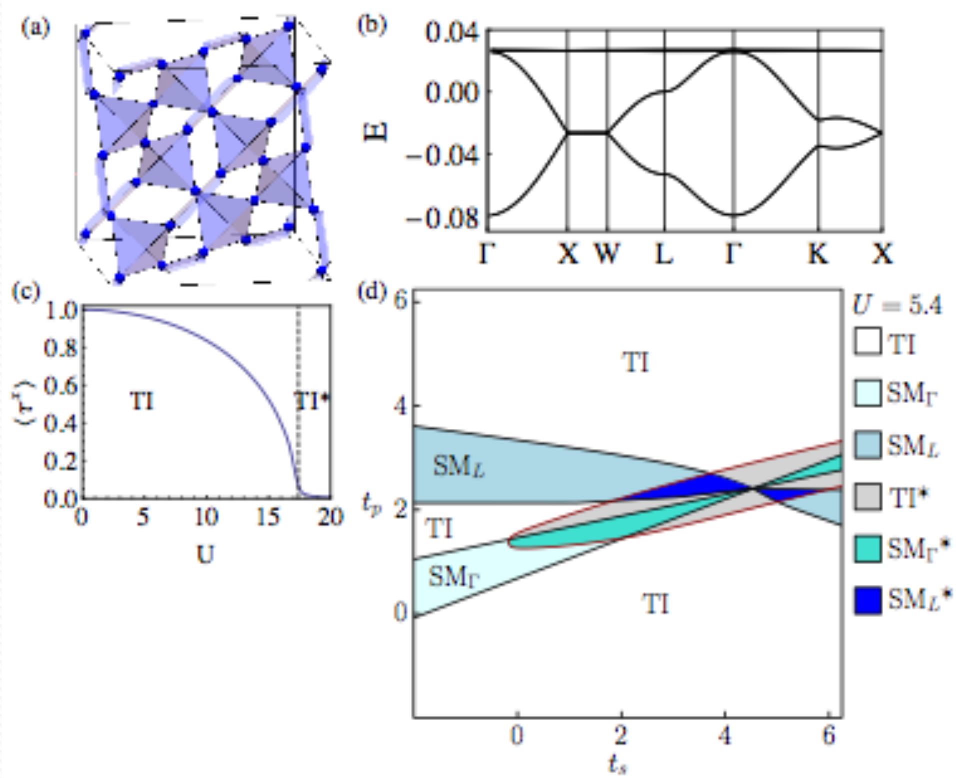
Mean-Field Theory for TI*: Phase Diagram and Braiding

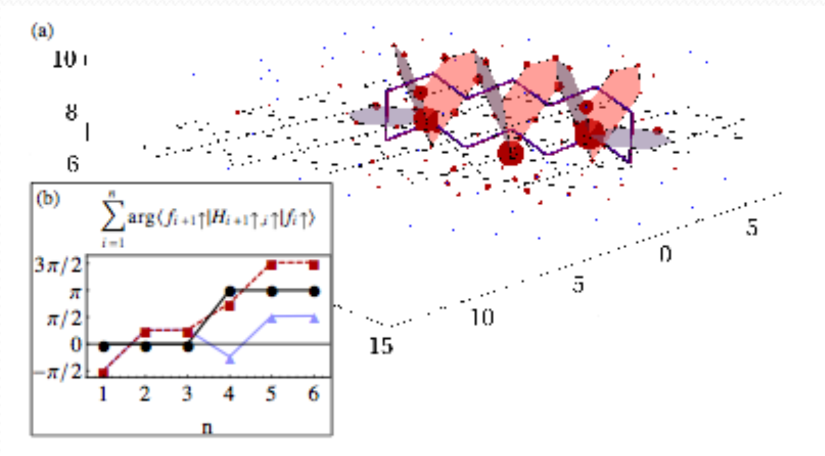
J. Maciejko, V. Chua, GAF PRL (2014)

$$H_{\mathrm{MF}} = \sum_{rr'}\sum_{\alpha\beta}\chi^{rr'}_{\alpha\beta}\sigma_{rr'} f^{\dagger}_{r\alpha} f_{r'\beta} + \sum_{rr'} J_{rr'}\sigma_{rr'}\tau^{x}_{r}\tau^{x}_{r'} + \frac{U}{4}\sum_{r}(\tau^{z}_{r}+1)$$

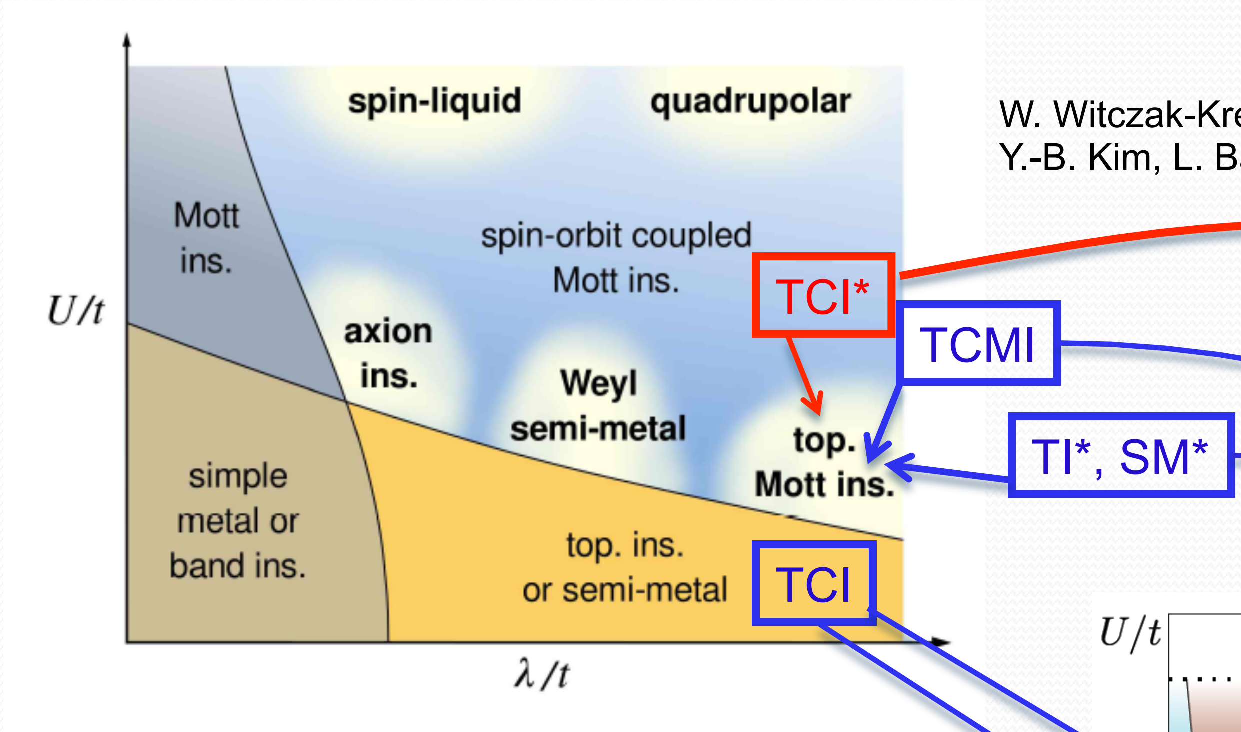
$$\chi^{rr'}_{\alpha\beta} = t^{rr'}_{\alpha\beta}\sigma_{rr'}\langle\tau^{x}_{r}\tau^{x}_{r'}\rangle_{\mathrm{MF}},$$

$$J_{rr'} = \sum_{\alpha\beta} t^{rr'}_{\alpha\beta}\sigma_{rr'}\langle f^{\dagger}_{r\alpha} f_{r'\beta}\rangle_{\mathrm{MF}}$$

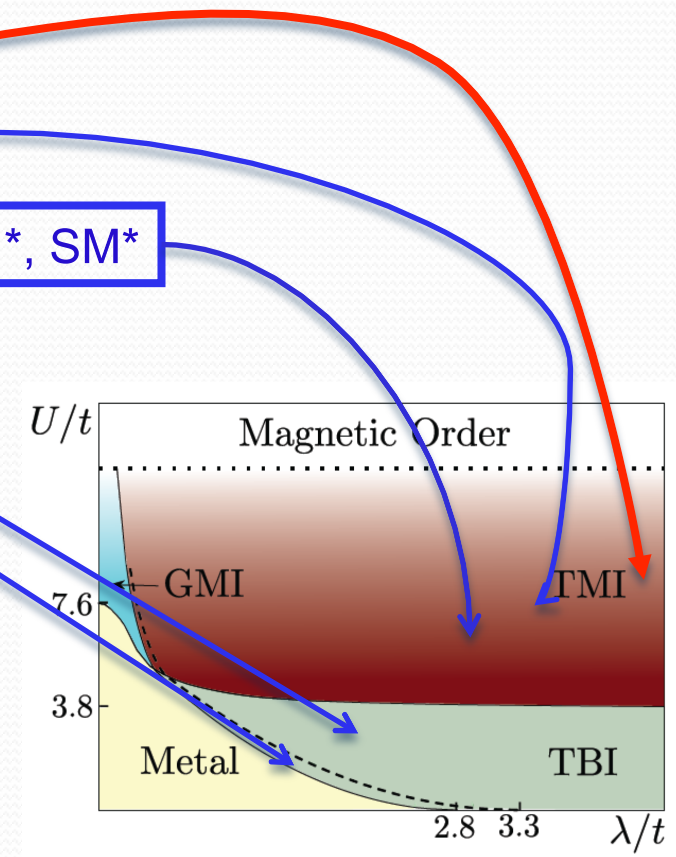




Schematic Phase Diagram



W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents (2014)



Pesin, Balents Nat. Phys. (2010)

- A_2Ir_2O7: M. Kargarian, GAF PRL (2013)
  Topological crystalline insulator, TCMI 4-2-2
- A_2Ir_2O7: J. Maciejko, V. Chua, GAF PRL (2014)
  TI* 4-2-2, SM* 2-4-2

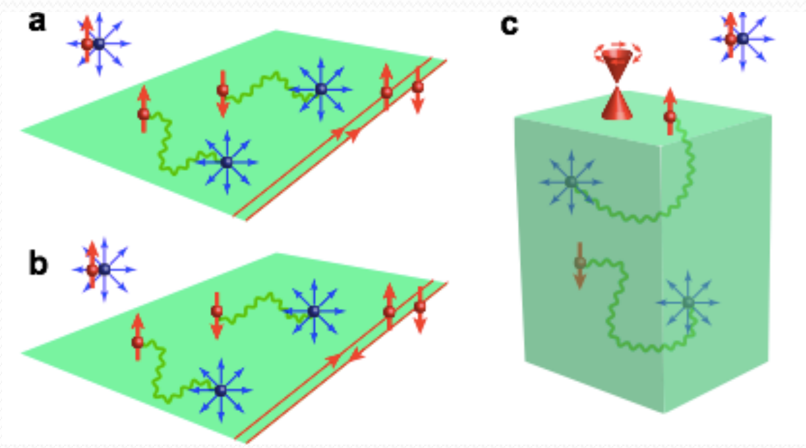
Summary of the Properties of Fractionalized Topological Phases

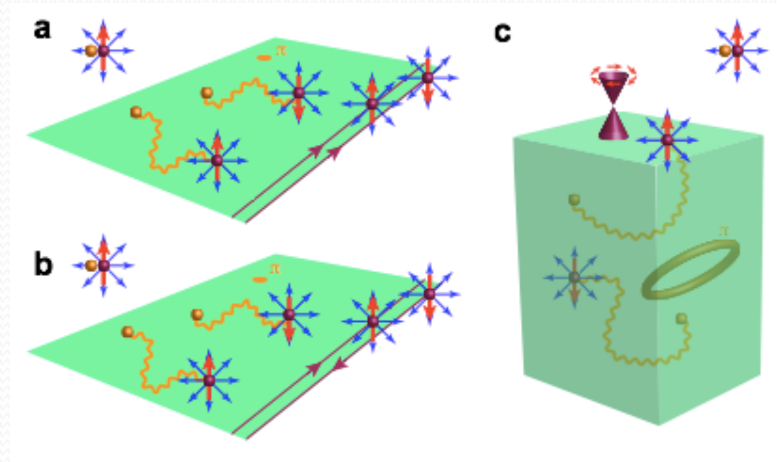
parent state	fractionalized state	d	symmetry	EM response	spin response	thermal response
CI	CSL	2		$\sigma_{xy}=0$	$\sigma^s_{xy}=0$	$\kappa_{xy}/T=2$
QSH	fractionalized QSH	2	TRS	$\sigma_{xy}=0$	$\sigma^s_{xy}=0$	$\kappa_{xy}/T=0$
TI/WTI	TMI/WTMI	3	TRS	$\theta=0$		$\theta_{\text{grav}}=\pi$ (TMI)
TCI	TCMI	3	mirror			
CI	CI*	2		$\sigma_{xy}=2$	$\sigma^s_{xy}=0$	$\kappa_{xy}/T=2$
QSH	QSH*	2	TRS	$\sigma_{xy}=0$	$\sigma^s_{xy}=2$	$\kappa_{xy}/T=0$
TI	TI*	3	TRS	$\theta=\pi$		$\theta_{\text{grav}}=\pi$
CI	FCI	2		$\sigma_{xy}=1/3$	$\sigma^s_{xy}=0$	$\kappa_{xy}/T=1$
QSH	FQSH	2	TRS	$\sigma_{xy}=0$	$\sigma^s_{xy}=2/3$	$\kappa_{xy}/T=0$
TI	FTI	3	TRS	$\theta=\pi/3$		$\theta_{\text{grav}}=\pi$

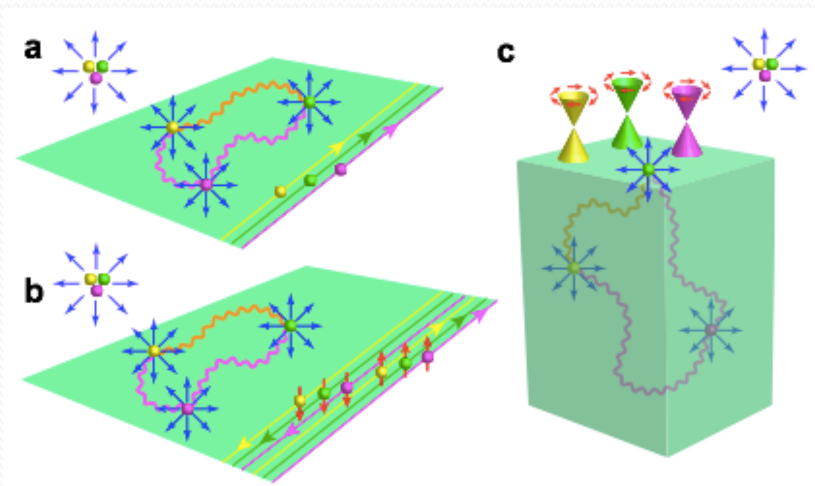
TABLE I. Summary of the d-dimensional fractionalized topological insulators discussed in this article and their universal response properties (acronyms are defined in the main text). σ_{xy}: Hall conductance in units of e^2/h; θ: coefficient of the $\boldsymbol{E}\cdot\boldsymbol{B}$ term in units of $e^2/2\pi h$; σ^s_{xy}: spin Hall conductance in units of $e/4\pi$ for a system with conserved z component of spin; κ_{xy}/T: thermal Hall conductance divided by the temperature, in units of $\pi^2 k_B^2/3h$; θ_{grav}: coefficient of the gravitational equivalent of the $\boldsymbol{E}\cdot\boldsymbol{B}$ term, responsible for a surface quantized thermal Hall effect.

J. Maciejko and GAF, *Nature Physics* Progress Article (Commissioned)

Visual Summary of Fractionalized Topological Phases







J. Maciejko and GAF, *Nature Physics* Progress Article (Commissioned)

Summary

- We have presented interacting lattice models that realize fractionalized topological phases.
- Transition metal oxides appear to be a promising venue for a variety of topological phases.
- Various ambitious classification schemes are currently under study for the interacting topological phases.