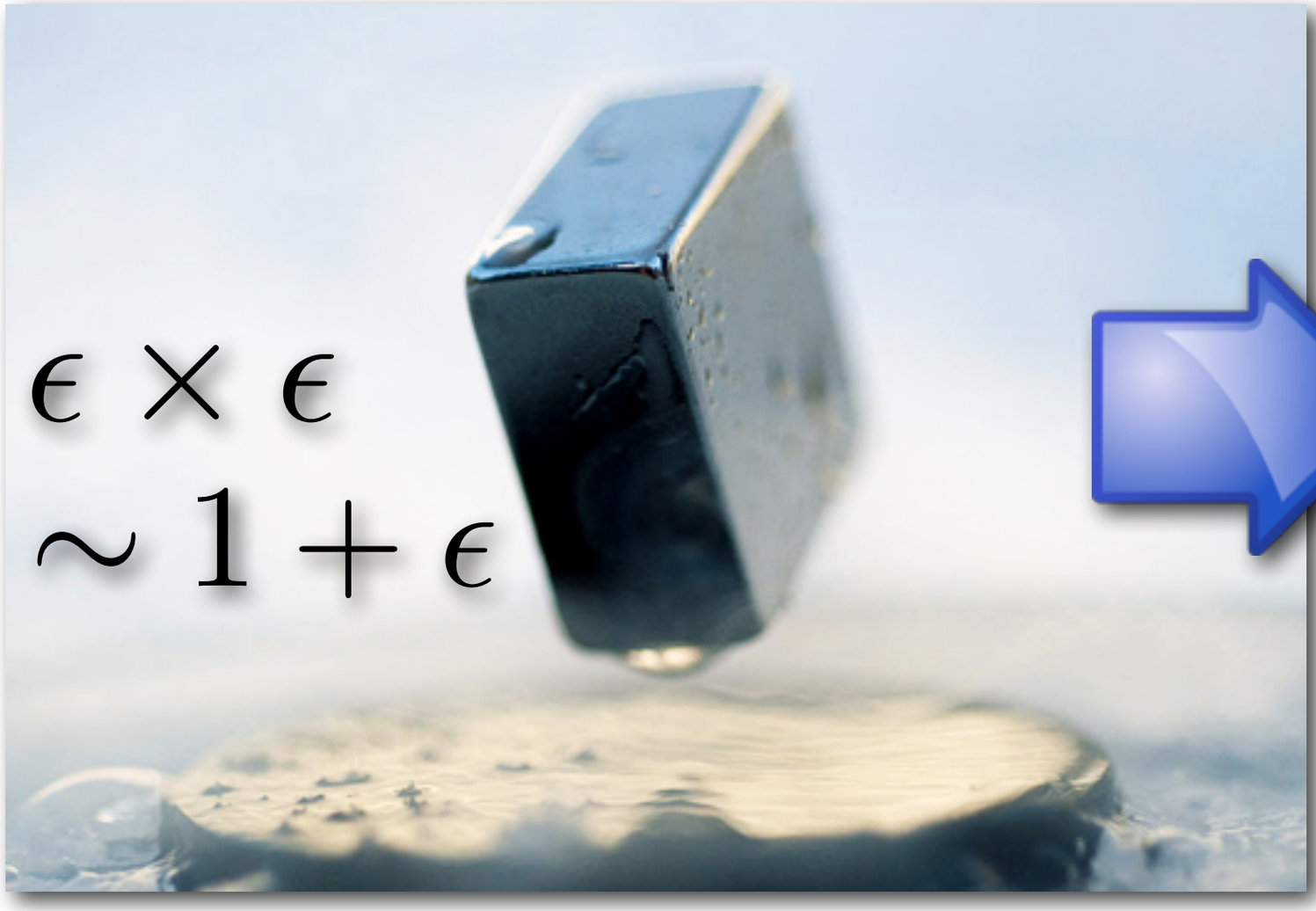
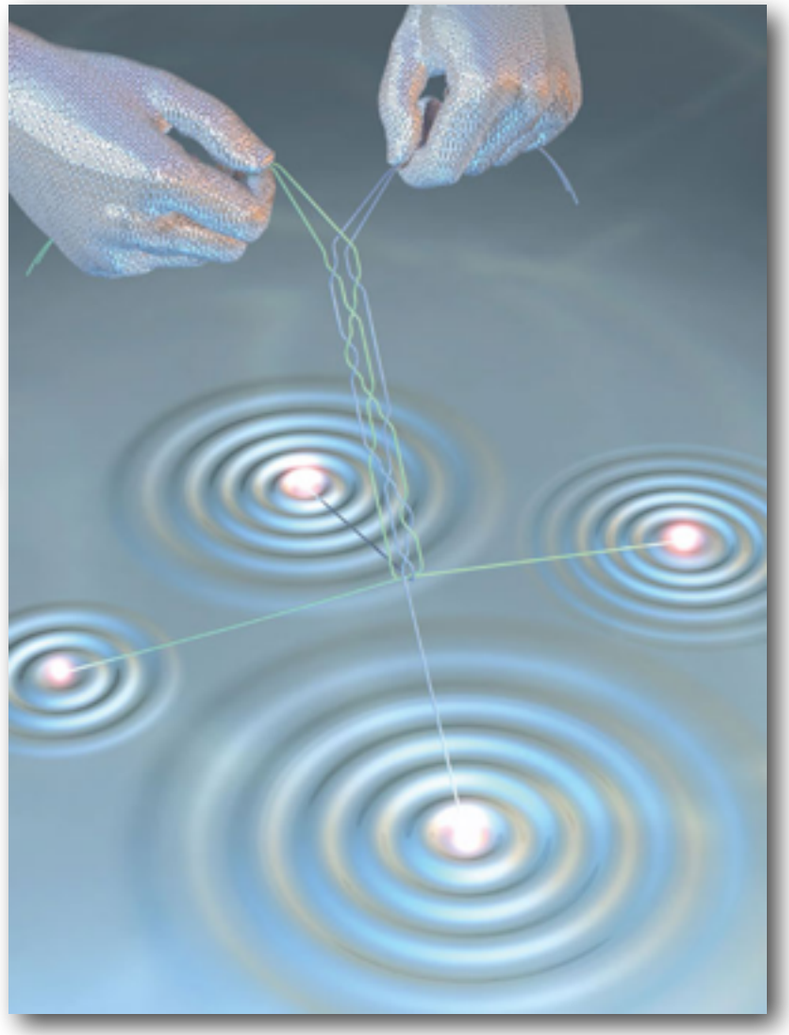
Designer non-Abelian anyons





Jason Alicea

Walter Burke Institute
for Theoretical Physics

Exchange statistics

1. Bosons/Fermions

(all fundamental particles)

$$\psi \rightarrow \pm\psi$$

Exchange statistics

I. Bosons/Fermions

(all fundamental particles)

$$\psi \rightarrow \pm\psi$$

II. "Anyons"

(*emergent* particles)

Abelian

Leinaas & Myrheim (1977)

$$\psi \rightarrow e^{i\theta}\psi$$

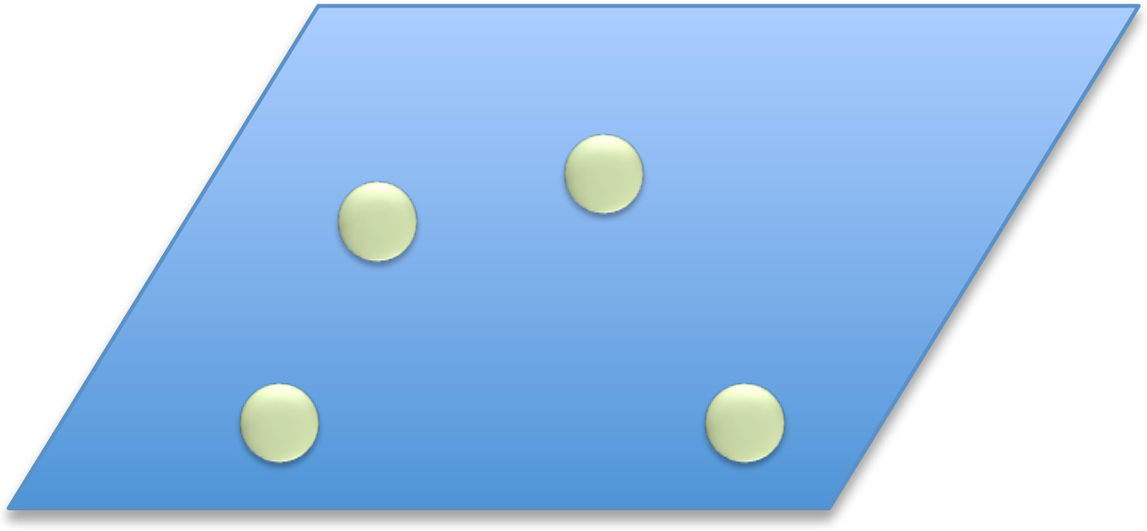
- Sequential exchanges commute
- Many experimental platforms!

Non-Abelian

Moore, Seiberg, Witten, Fredenhagen, Frohlich, Gabianni, Bais… (1988-)

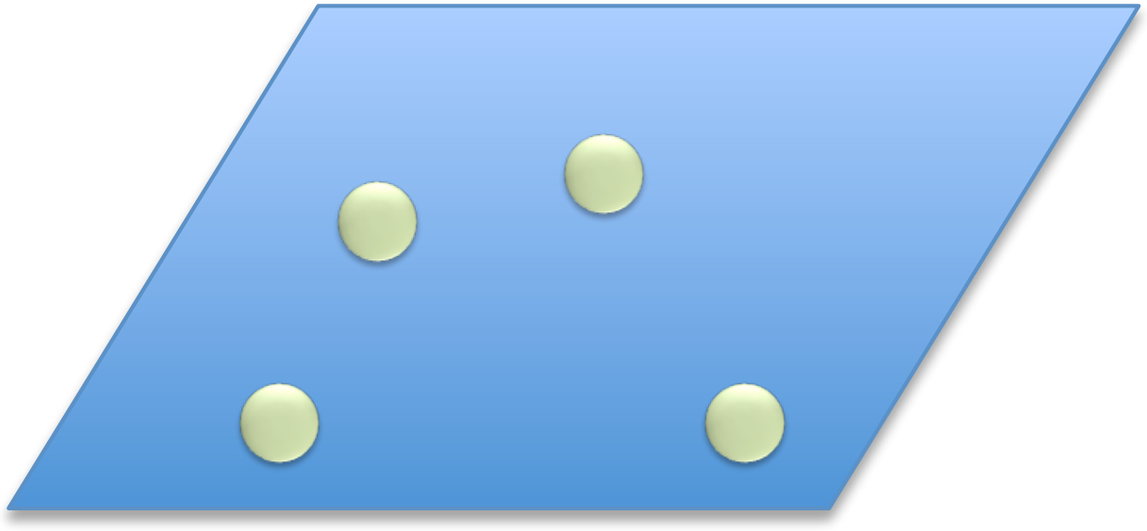
- Much more exotic and elusive

Non-Abelian anyons



$$\psi_a$$

Non-Abelian anyons



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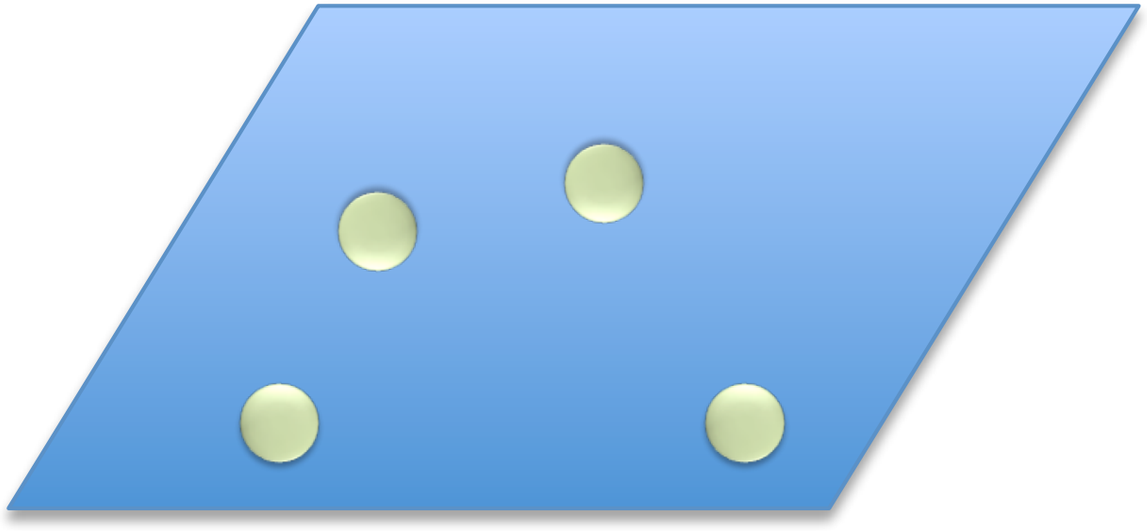
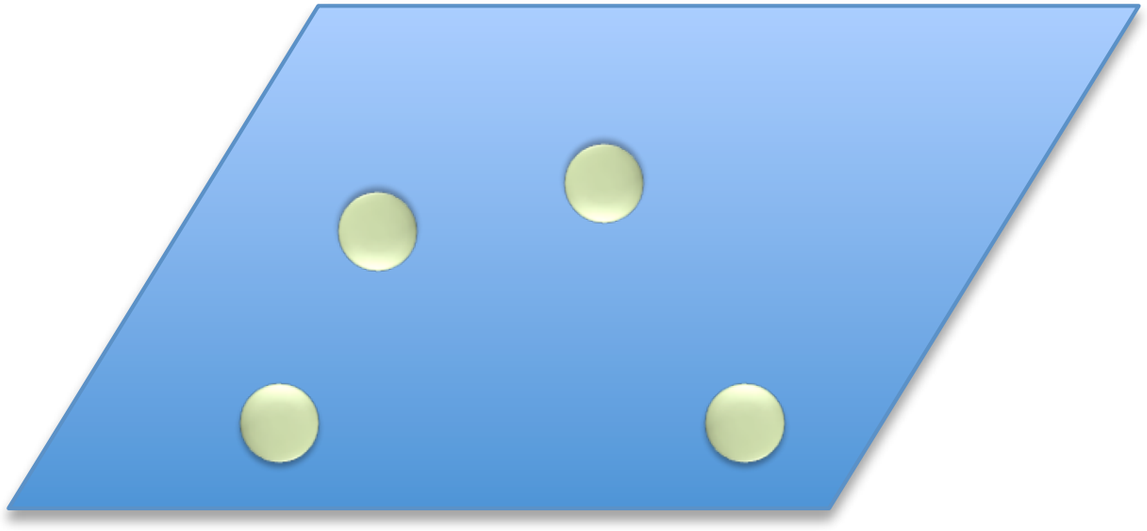


$$\psi_a \rightarrow U_{ab}\psi_b$$

Interesting for 2 reasons:

- Fundamental physics

Non-Abelian anyons



$$\psi_a \rightarrow U_{ab}\psi_b$$

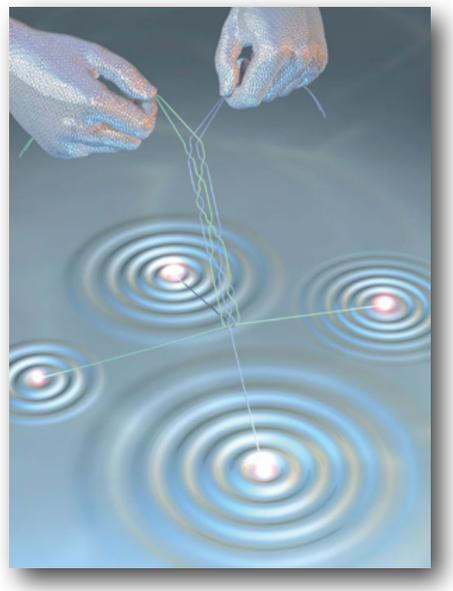
Qubits **Quantum gates**

Interesting for 2 reasons:

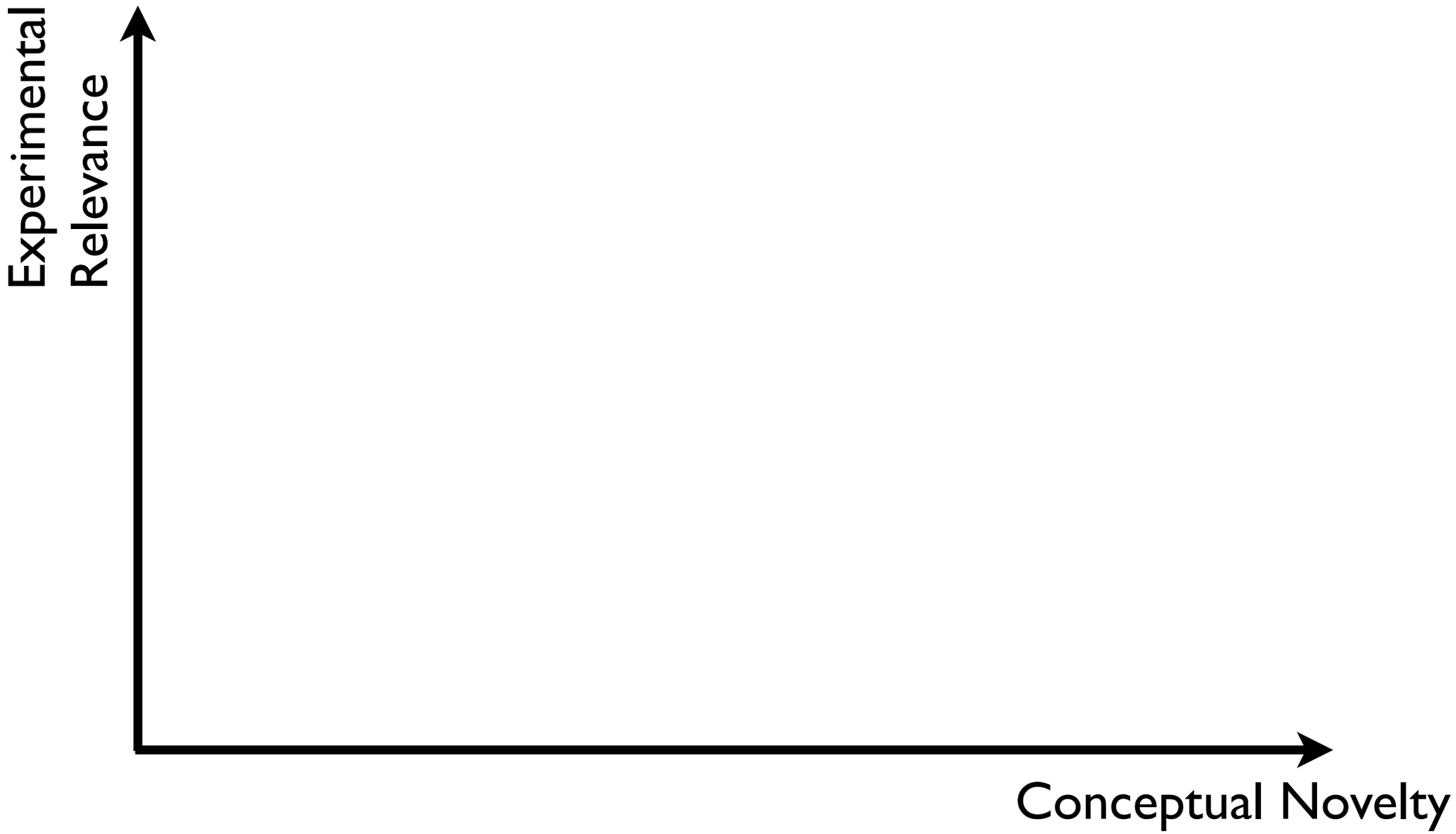
- Fundamental physics
- Decoherence-free quantum computation!

Kitaev (1997)
Nayak, Simon, Stern, Freedman, &
Das Sarma, RMP **80**, 1083 (2008)

"$3 million idea"



“Fisher plot”



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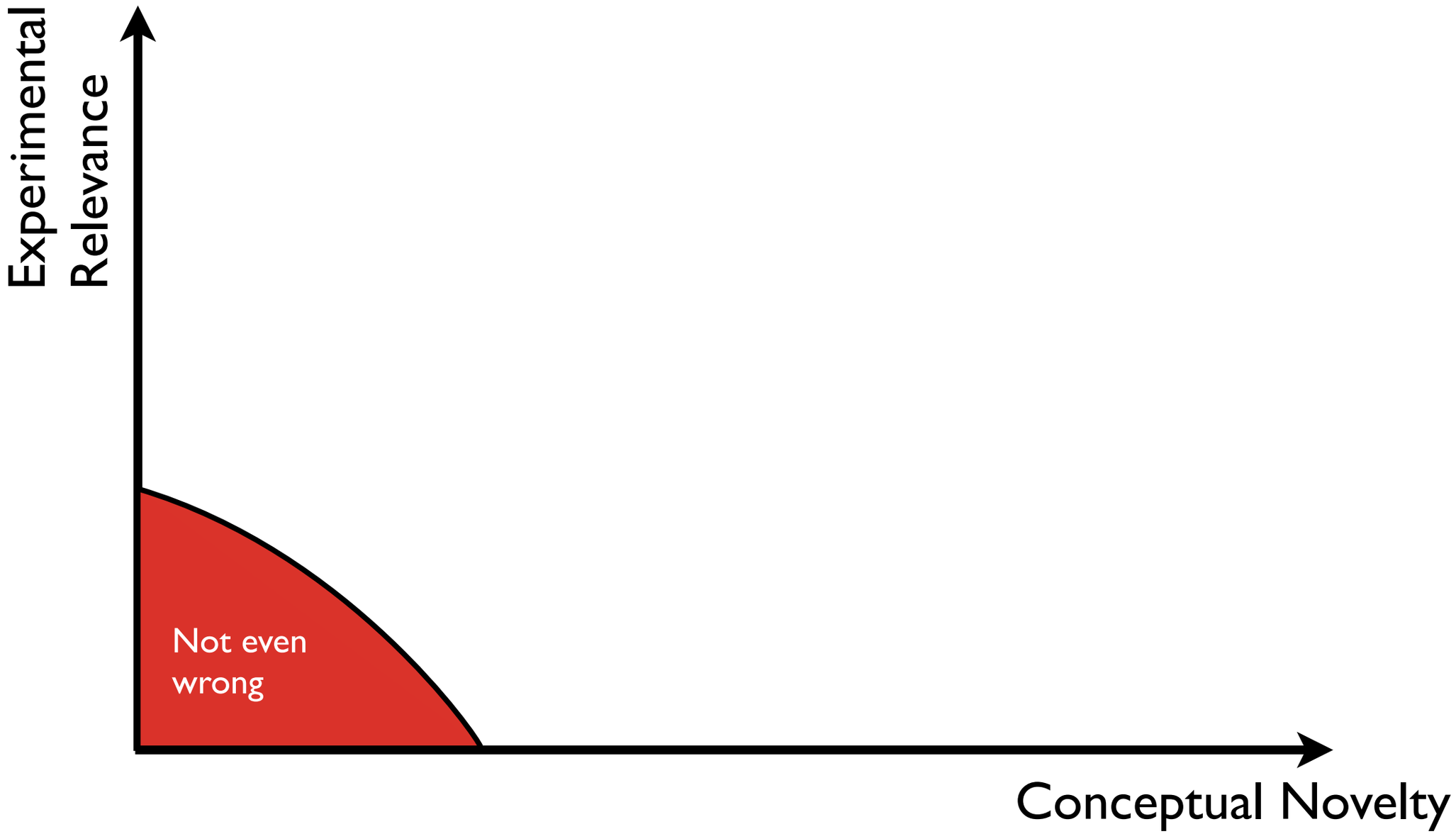
Experimental Relevance
Not even wrong
Conceptual Novelty

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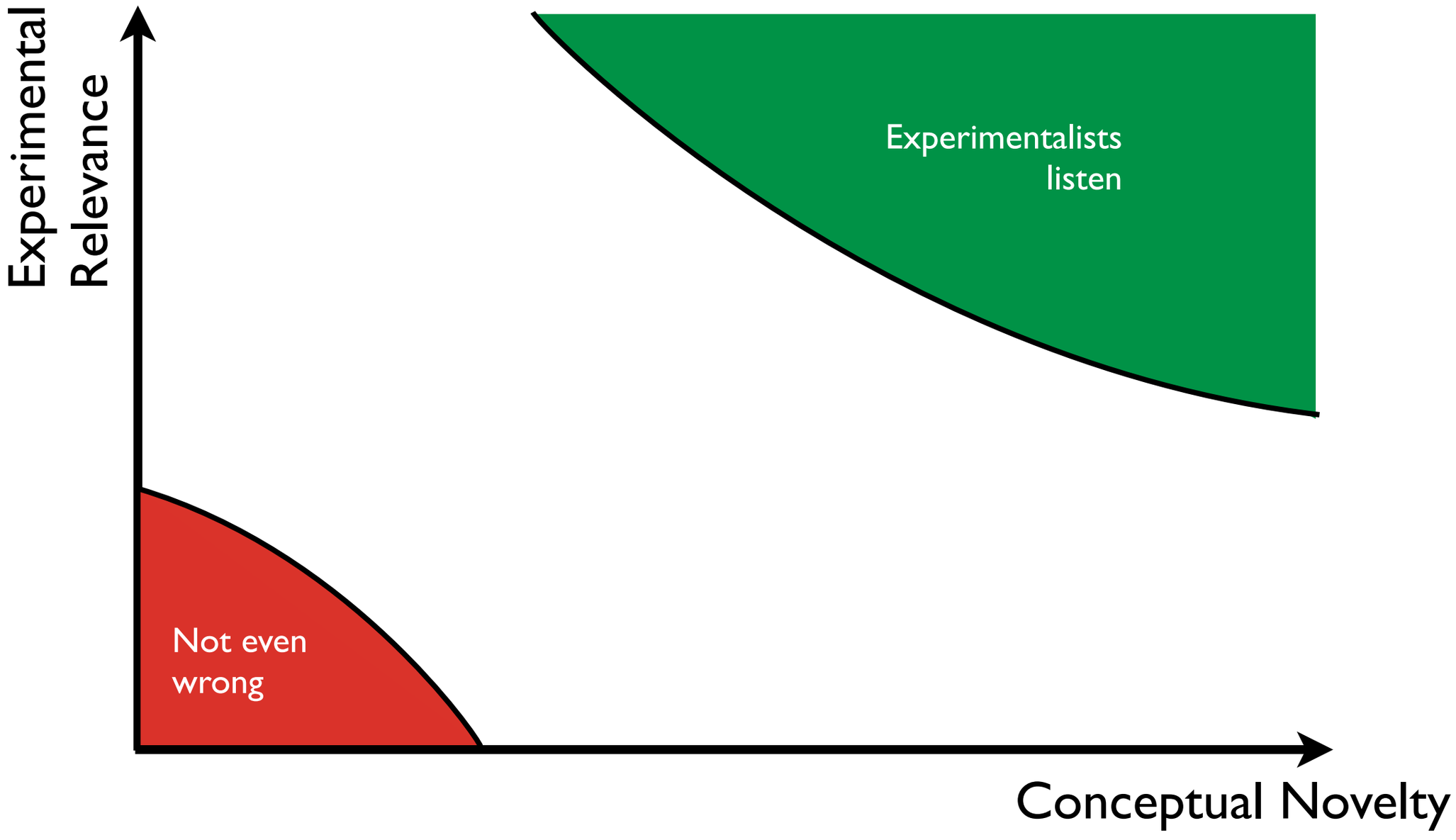
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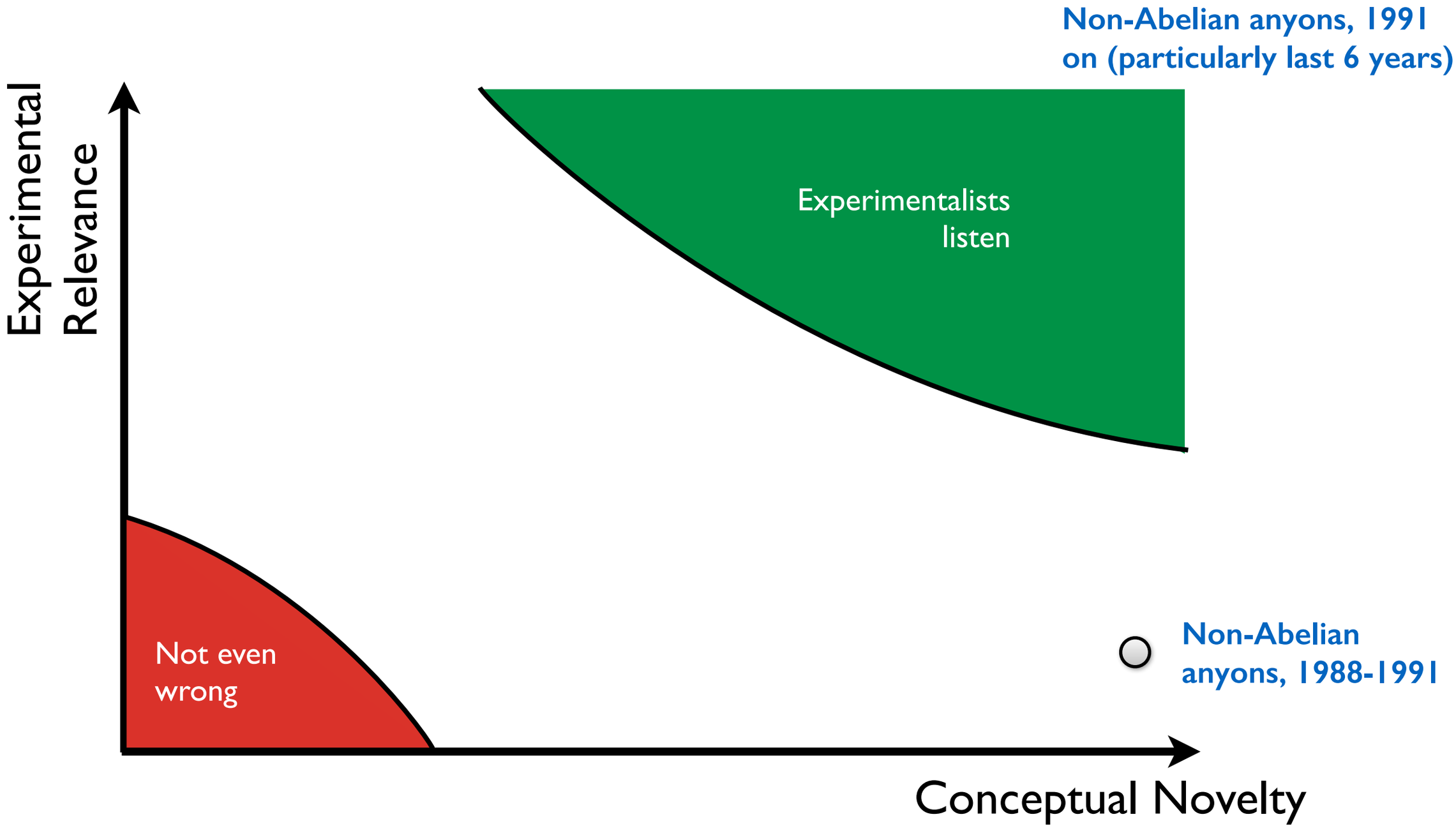
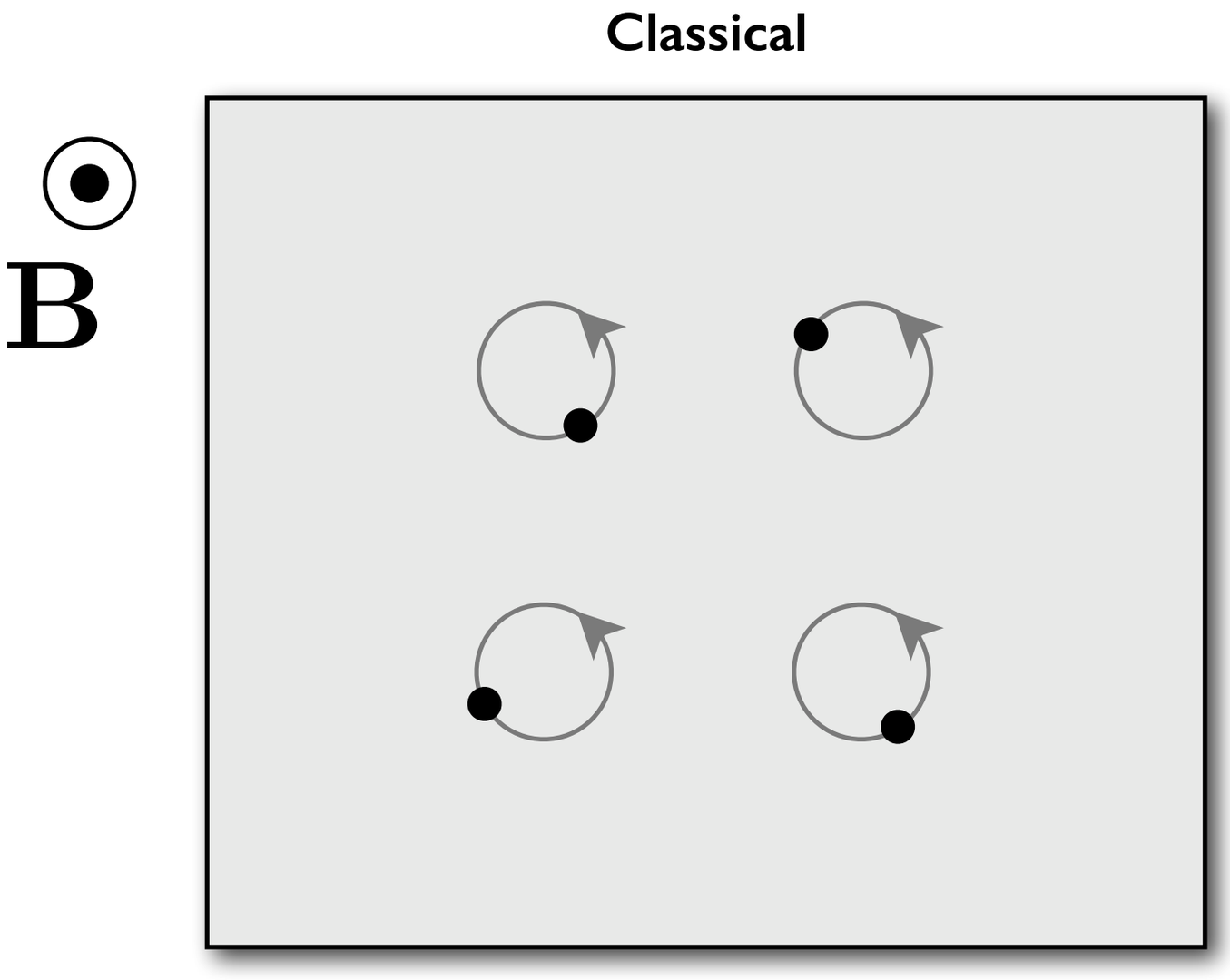

Non-Abelian anyons, 1991 on (particularly last 6 years)
Experimental Relevance
Experimentalists listen
Not even wrong
Non-Abelian anyons, 1988-1991
Conceptual Novelty

Experimentally vibrant field

Mathematical abstraction

Whirlwind quantum Hall overview



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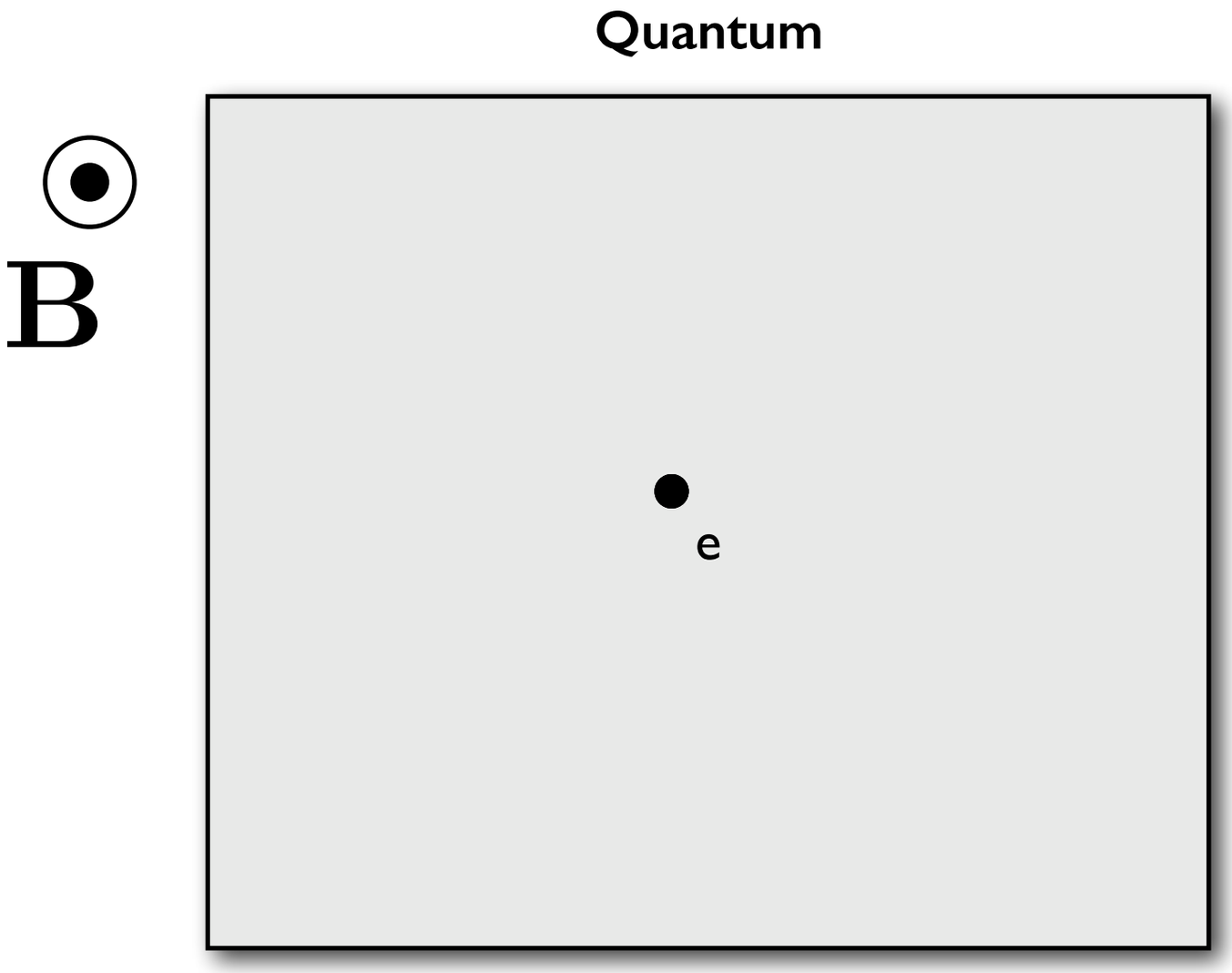
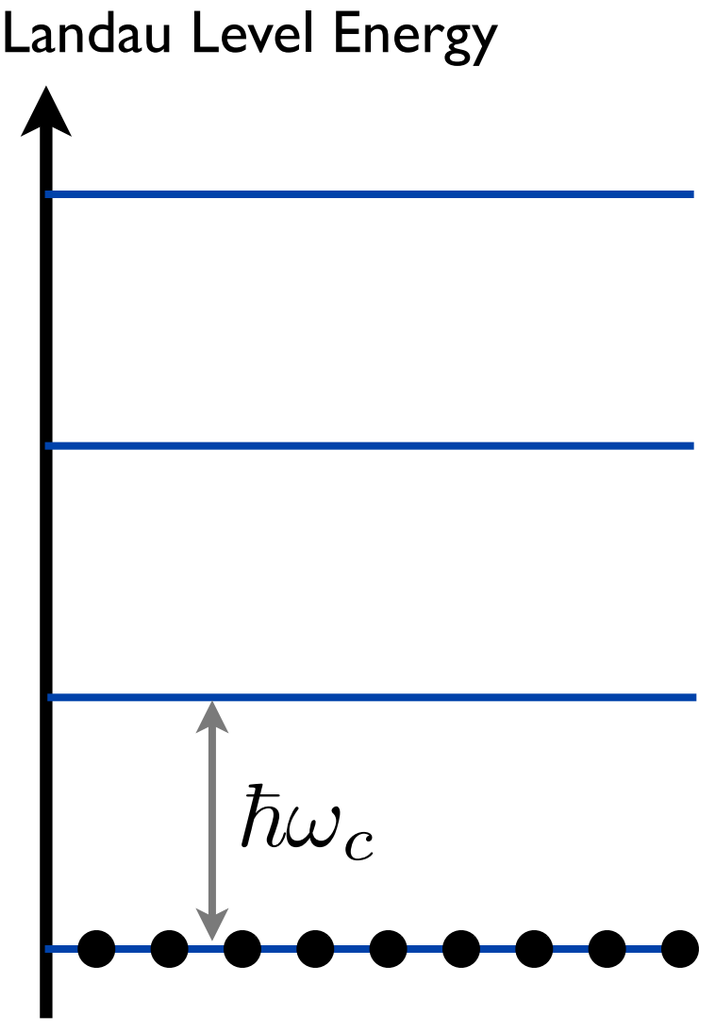
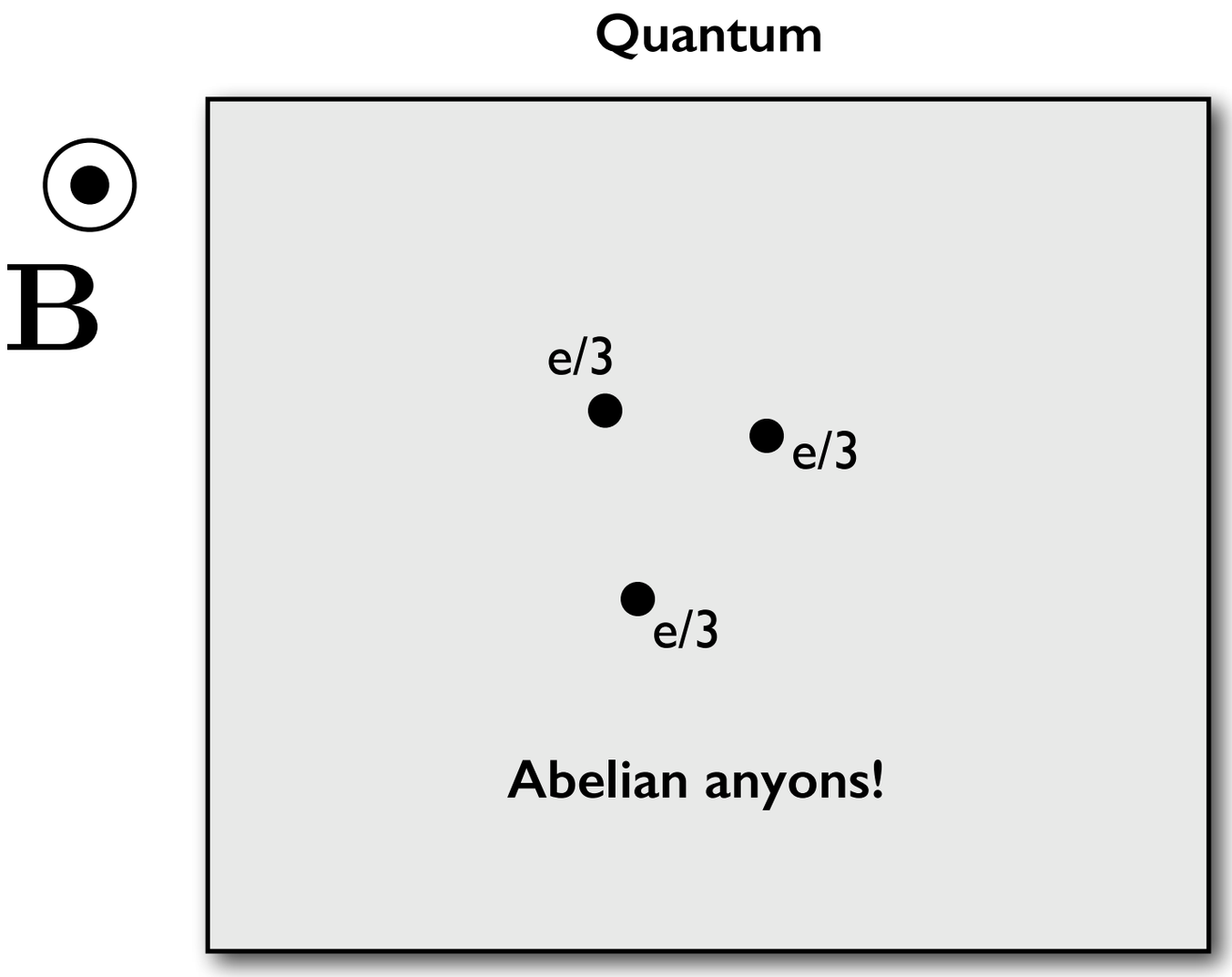
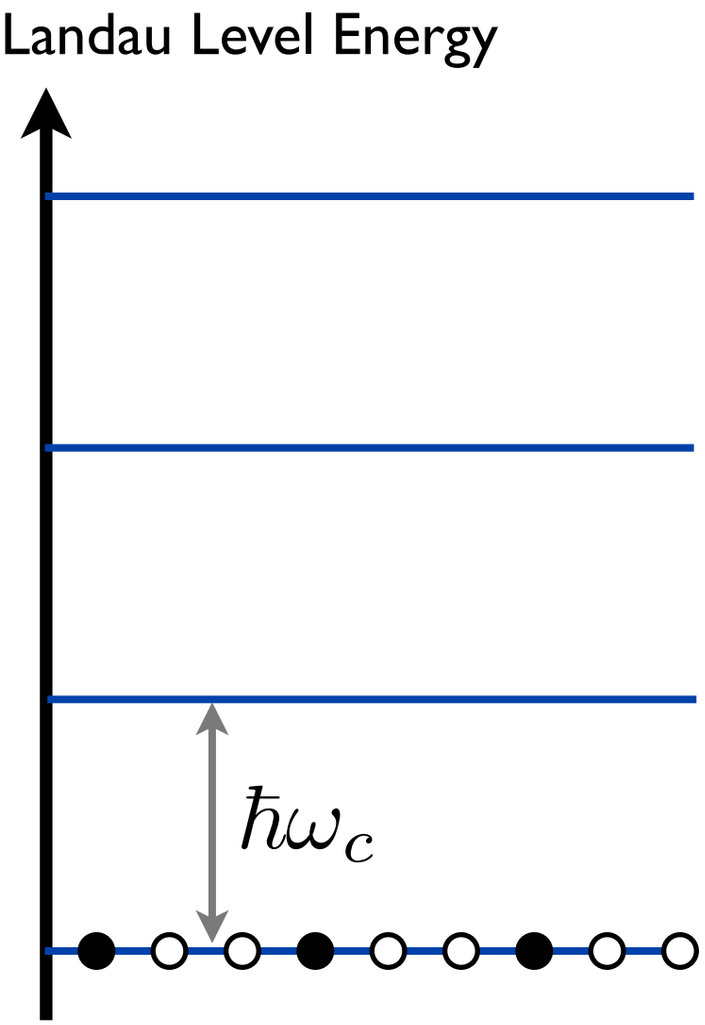




Physics depends sensitively on Landau level filling.

Whirlwind quantum Hall overview

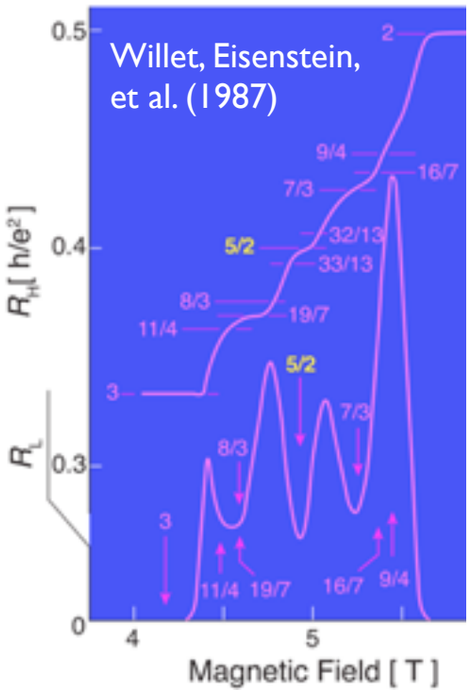




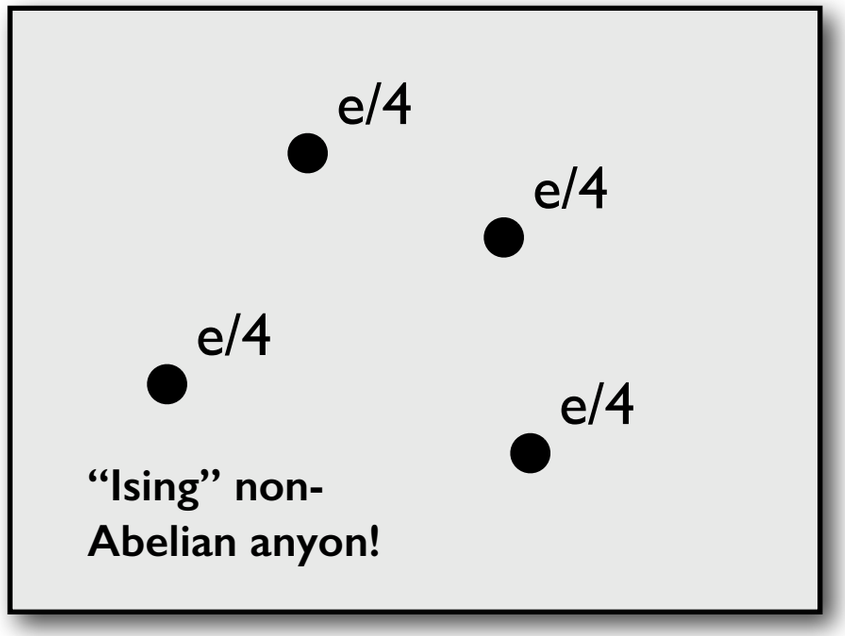
GaAs, graphene, oxide interfaces, CdTe,…

Physics depends sensitively on Landau level filling.

First proposed non-Abelian platforms



Moore-Read state



Moore & Read (1991)

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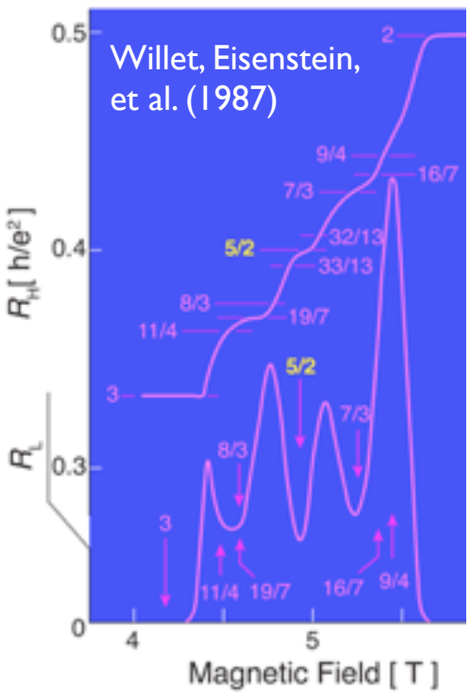

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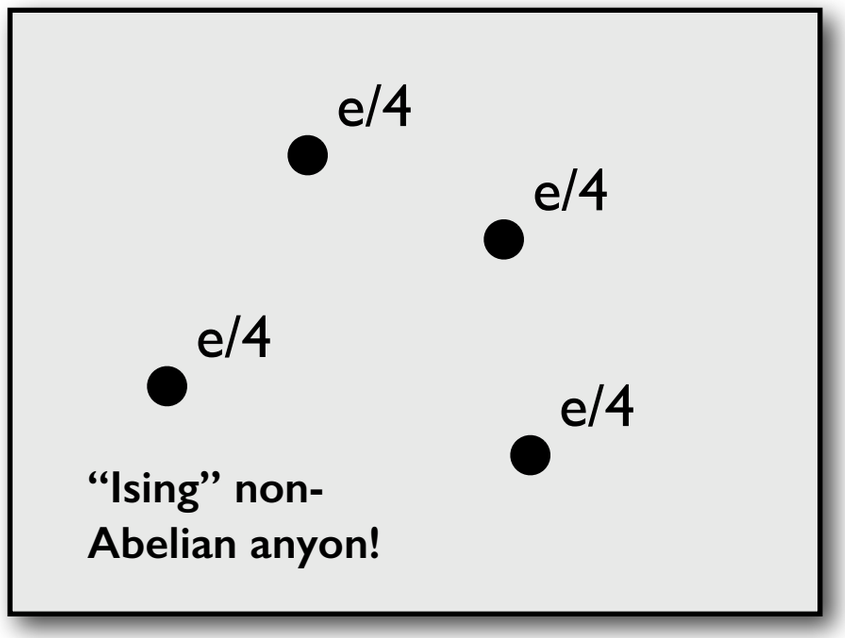
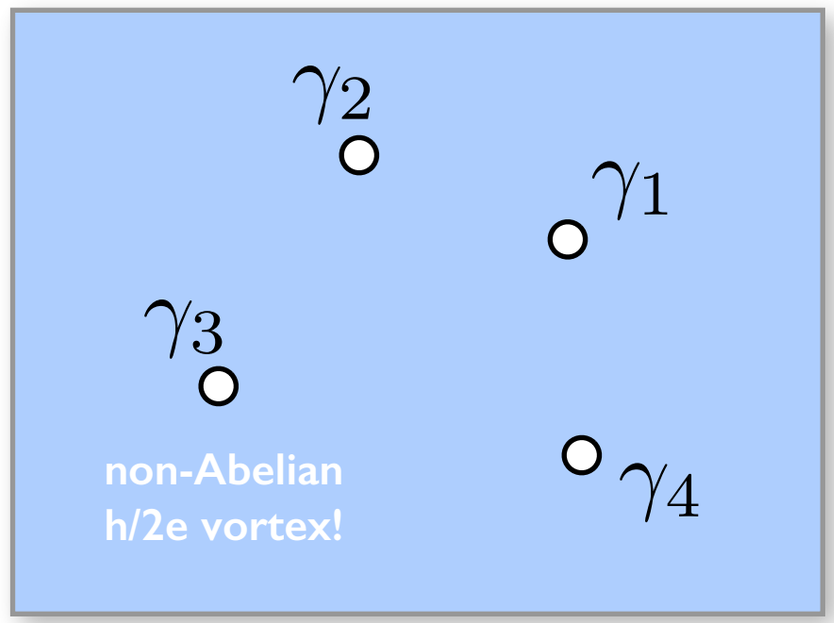


Moore & Read (1991)

~

2D "spinless" p+ip superconductor



Read & Green (2000); see also He-3
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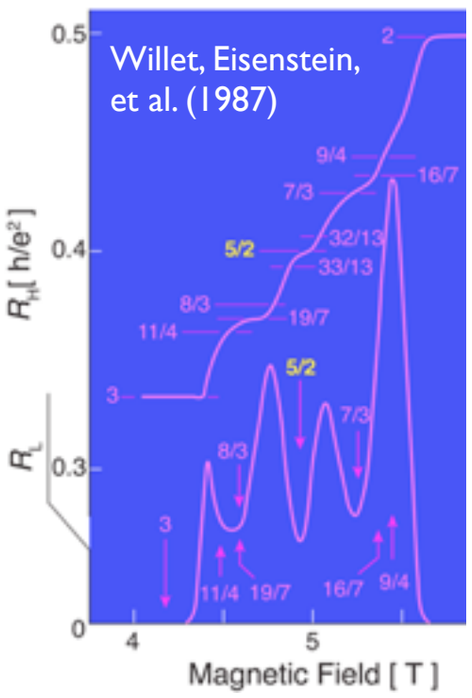

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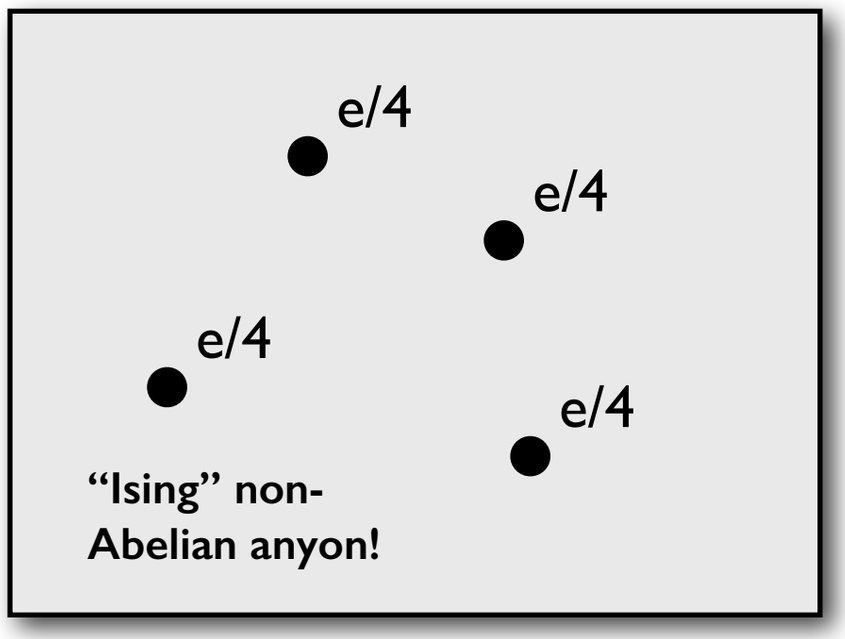

Moore & Read (1991)

$\sim$

2D "spinless" p+ip superconductor

Read & Green (2000); see also He-3 work by Volovik

Note: these are very different from Ettore Majorana's fermionic particles that are own antiparticles.

Degeneracy encoded in **Majorana zero-modes**

$$\gamma_i^\dagger = \gamma_i \qquad \gamma_i^2 = 1 \qquad \gamma_i \gamma_j = -\gamma_j \gamma_i$$

One Majorana = "half" a fermion

$$f_A = \gamma_1 + i\gamma_2$$

$$f_B = \gamma_3 + i\gamma_4$$

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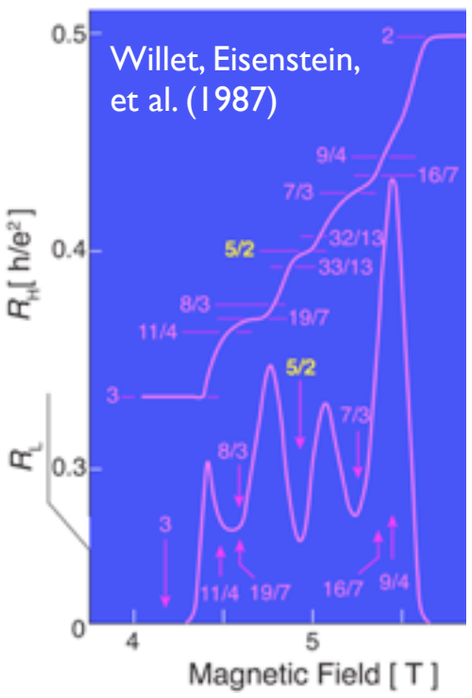

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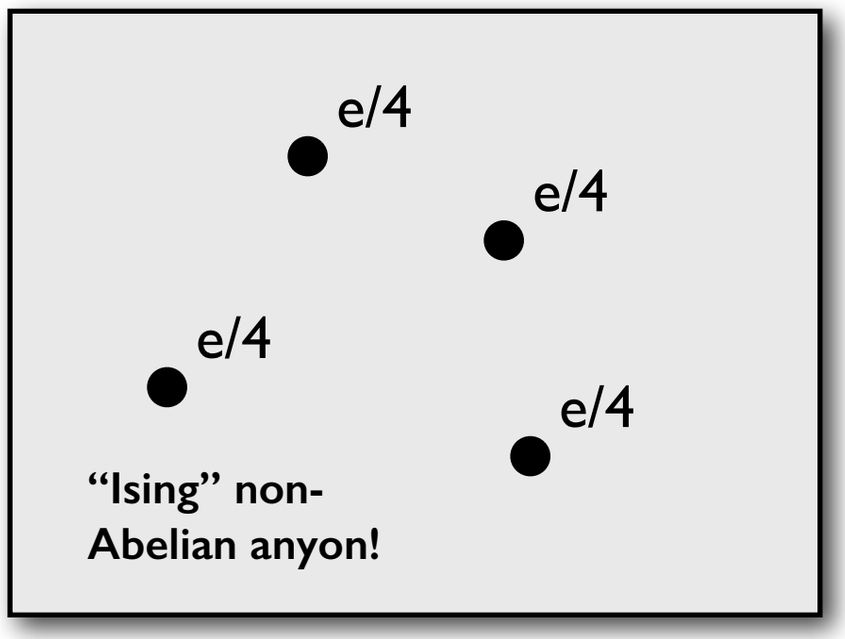

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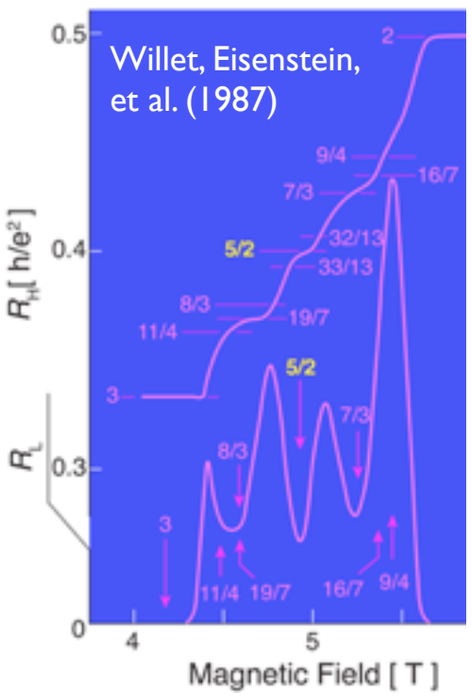

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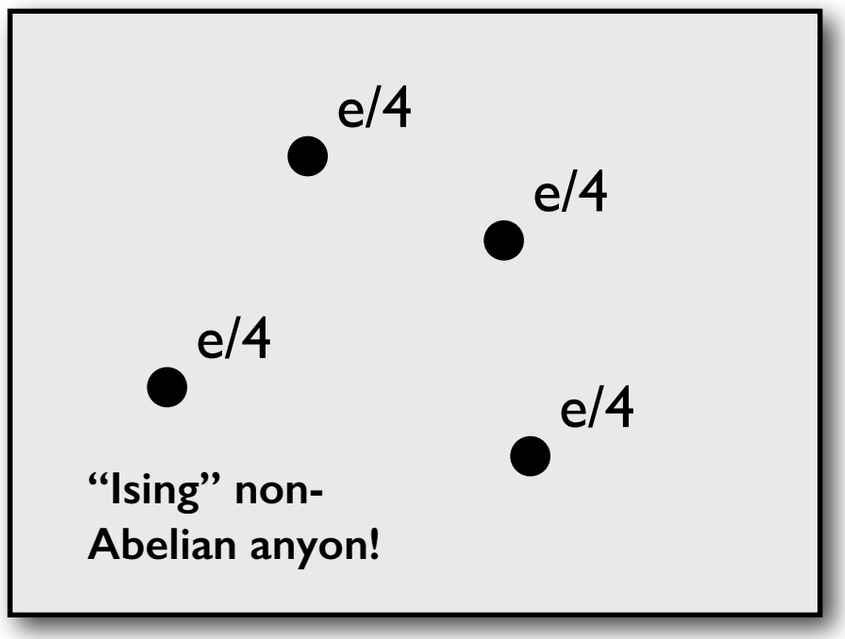

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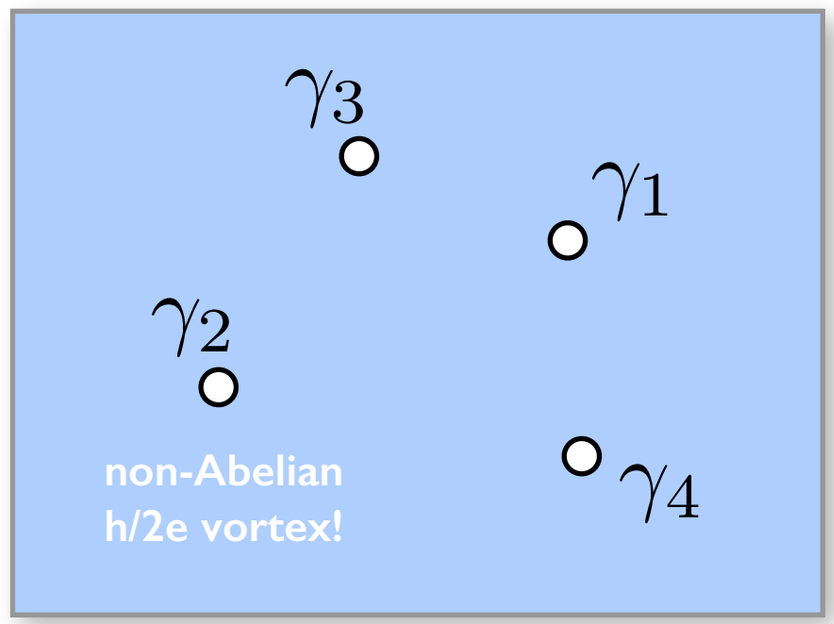

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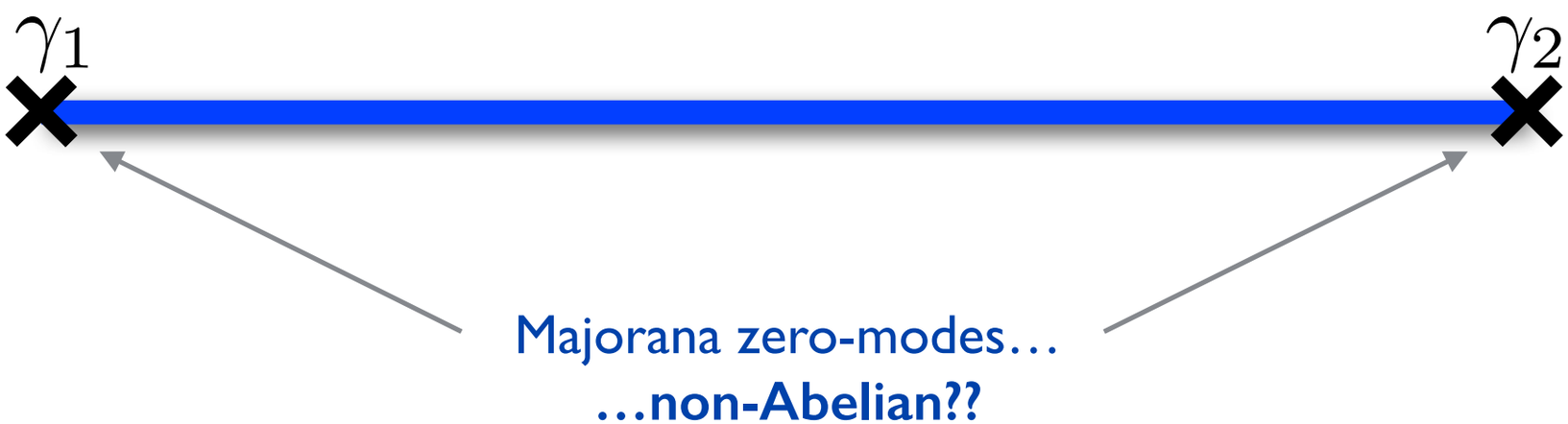

Kitaev (2001)

First proposed non-Abelian platforms

Historical view: exotic exchange statistics special to 2D world

Modern thinking: non-Abelian statistics possible in any dimension!

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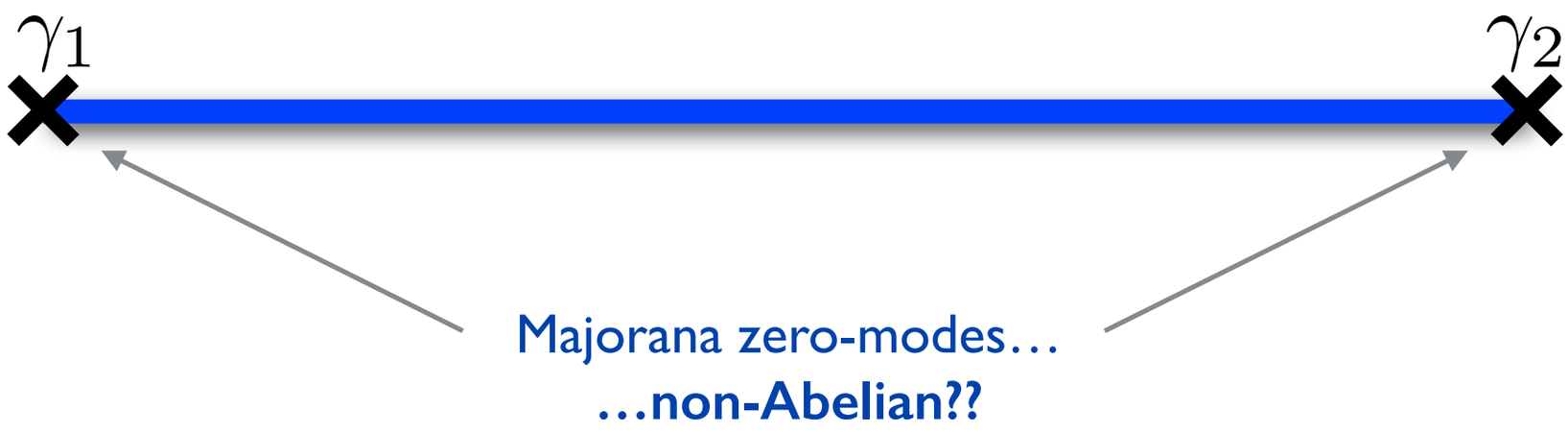

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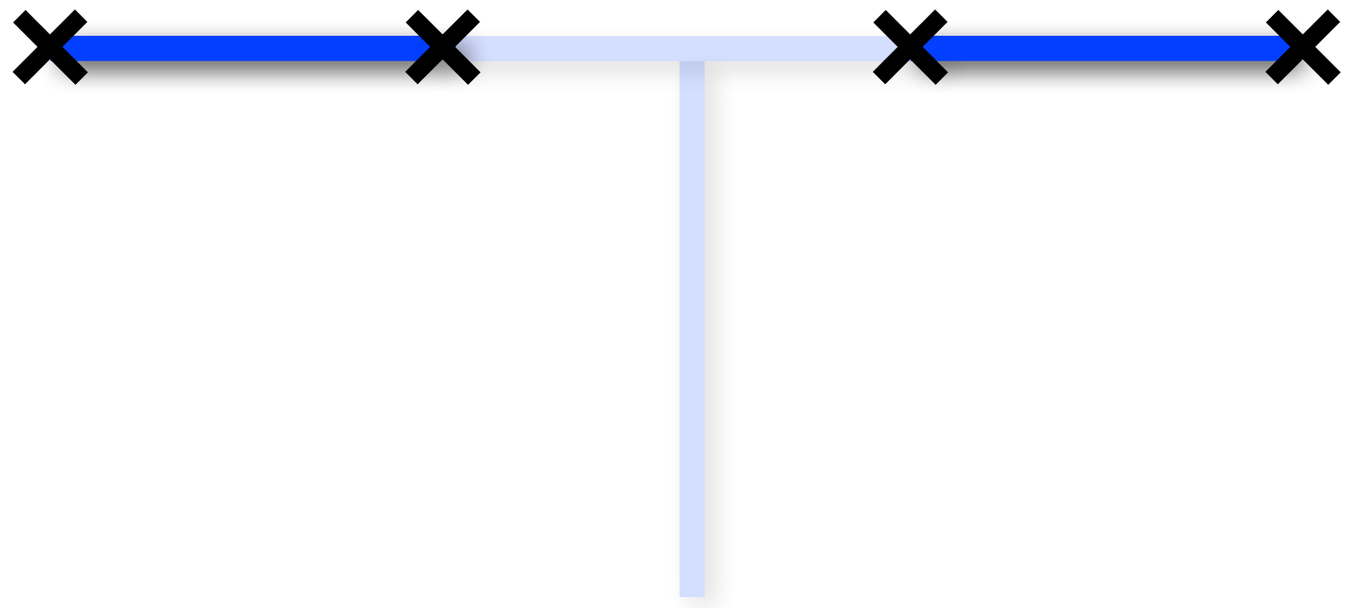

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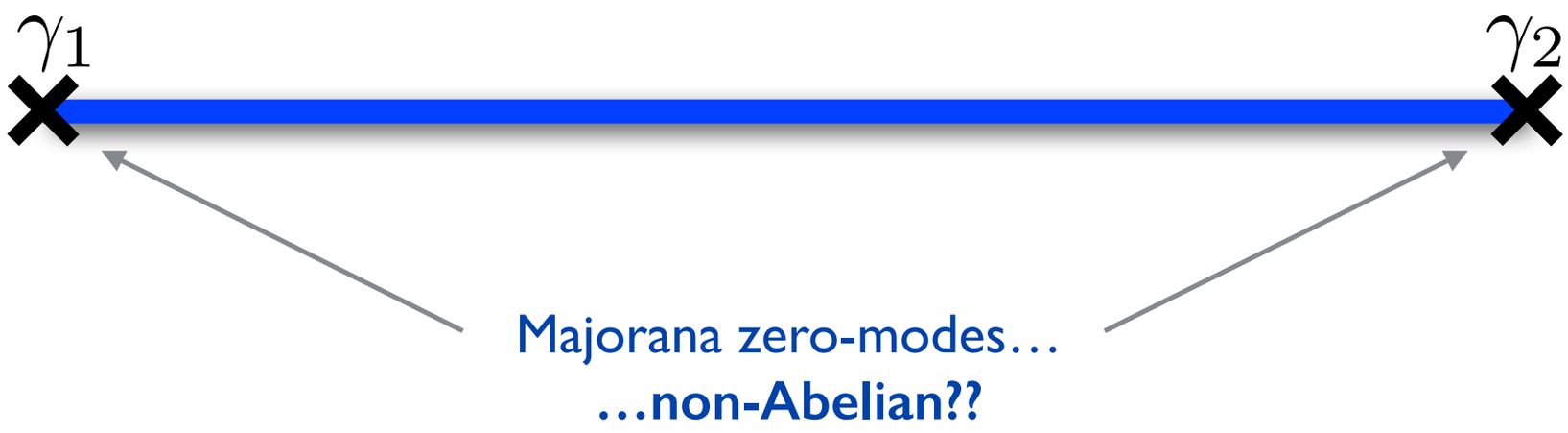

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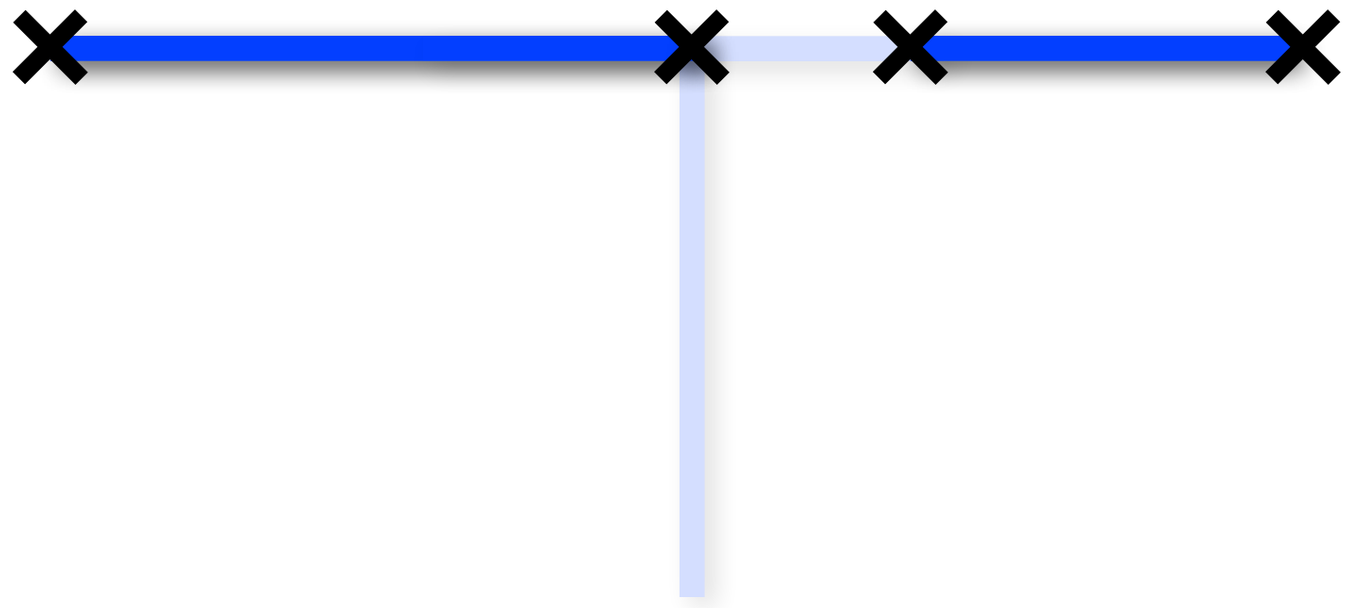

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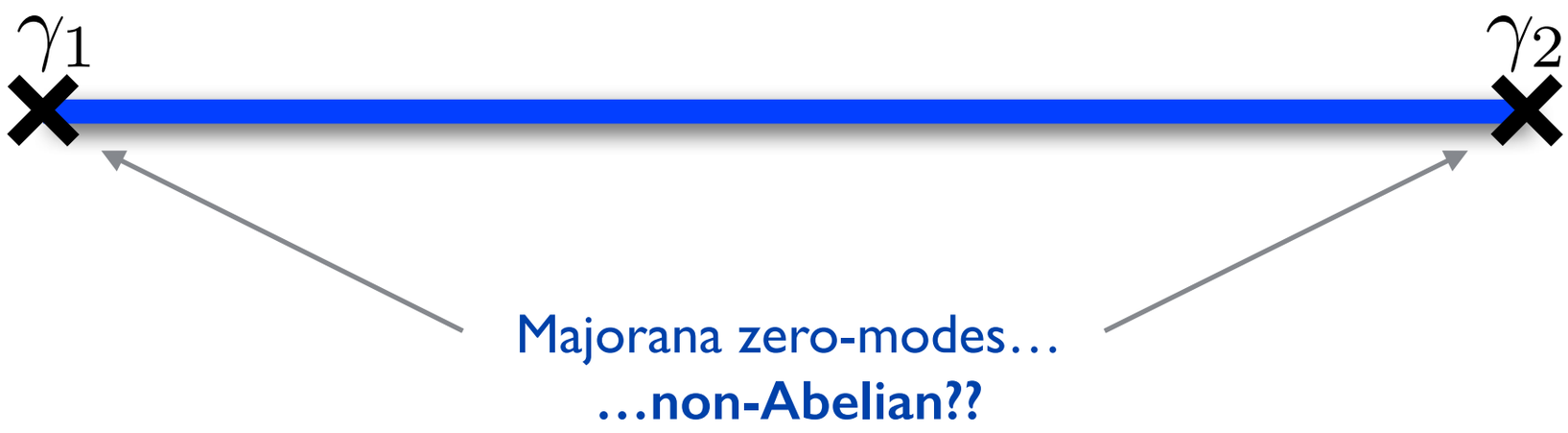

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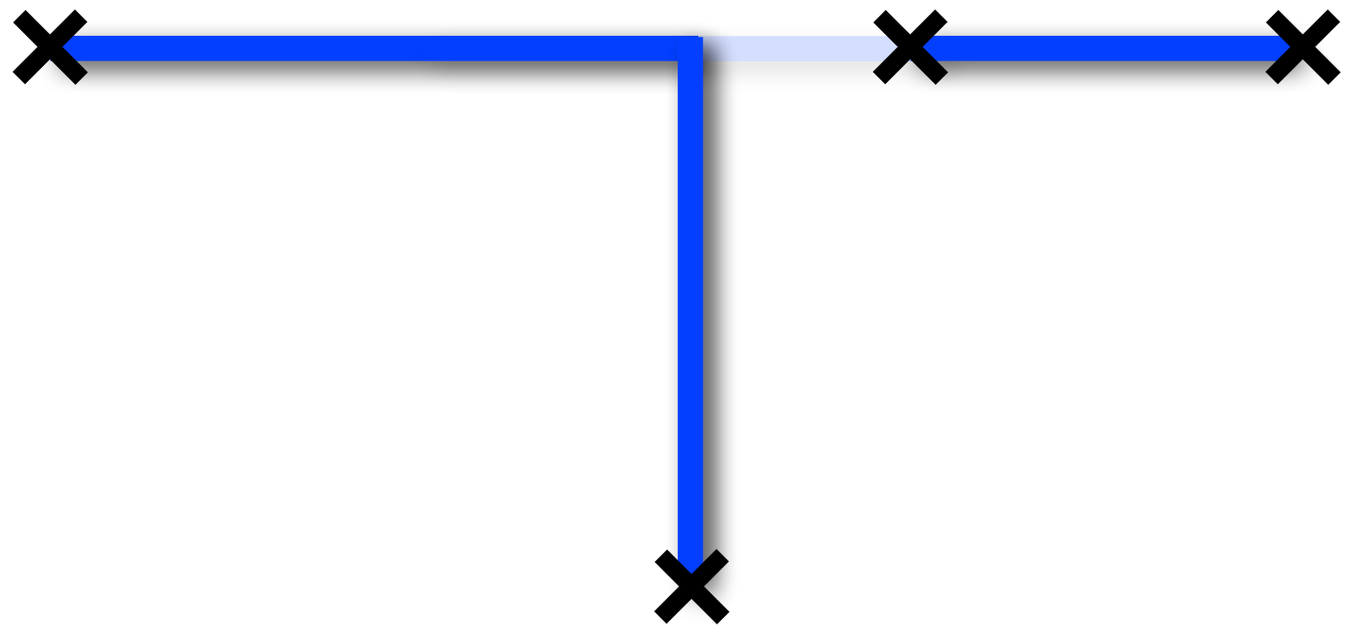

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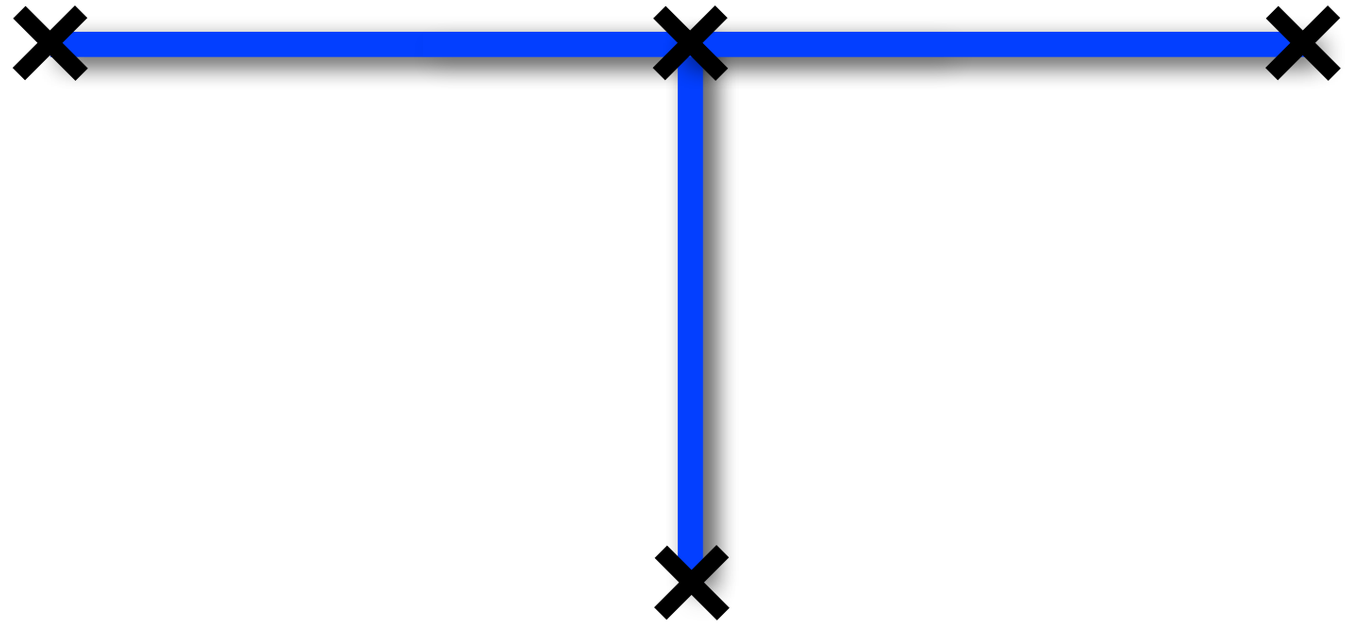

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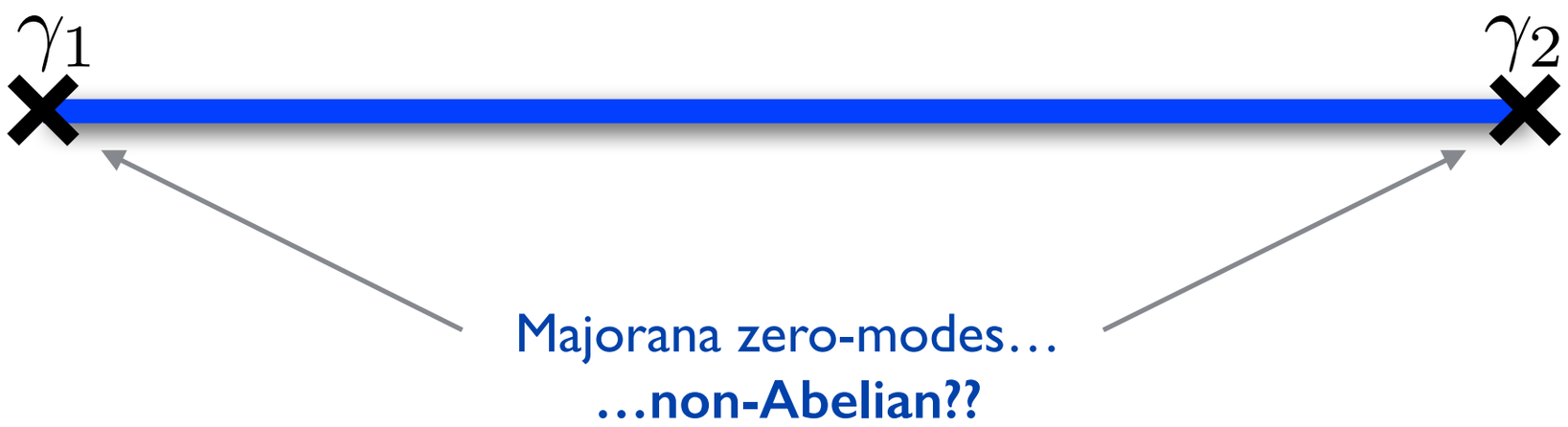

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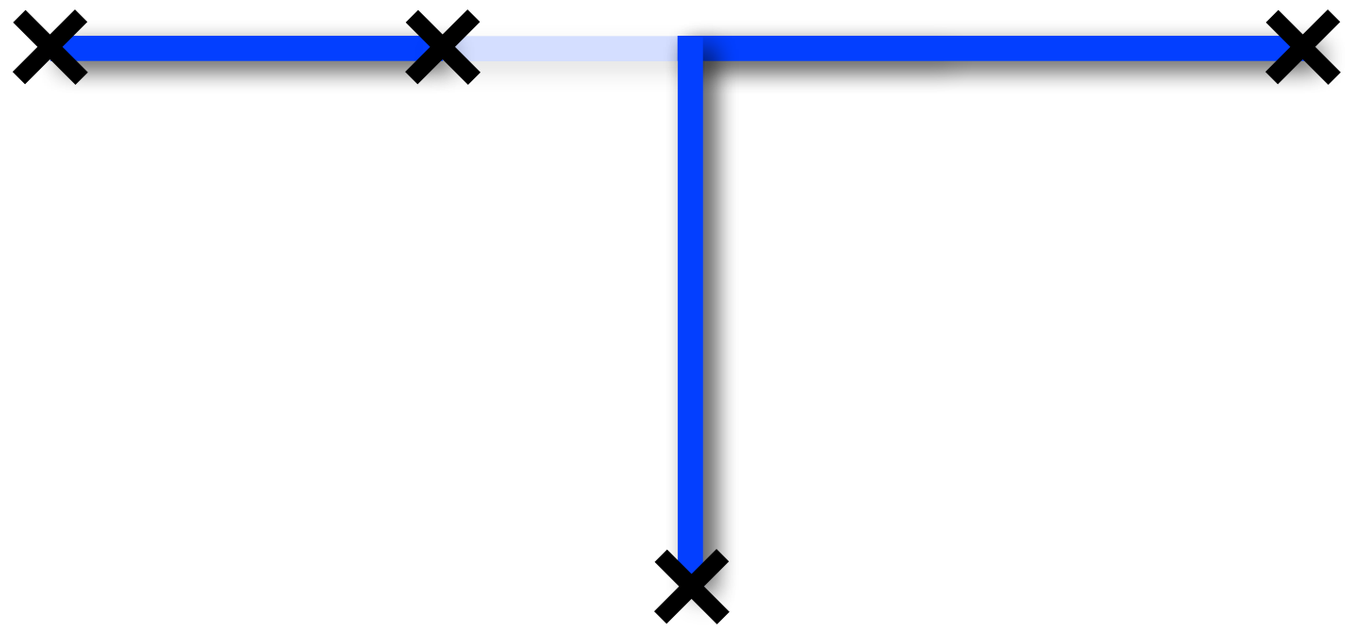

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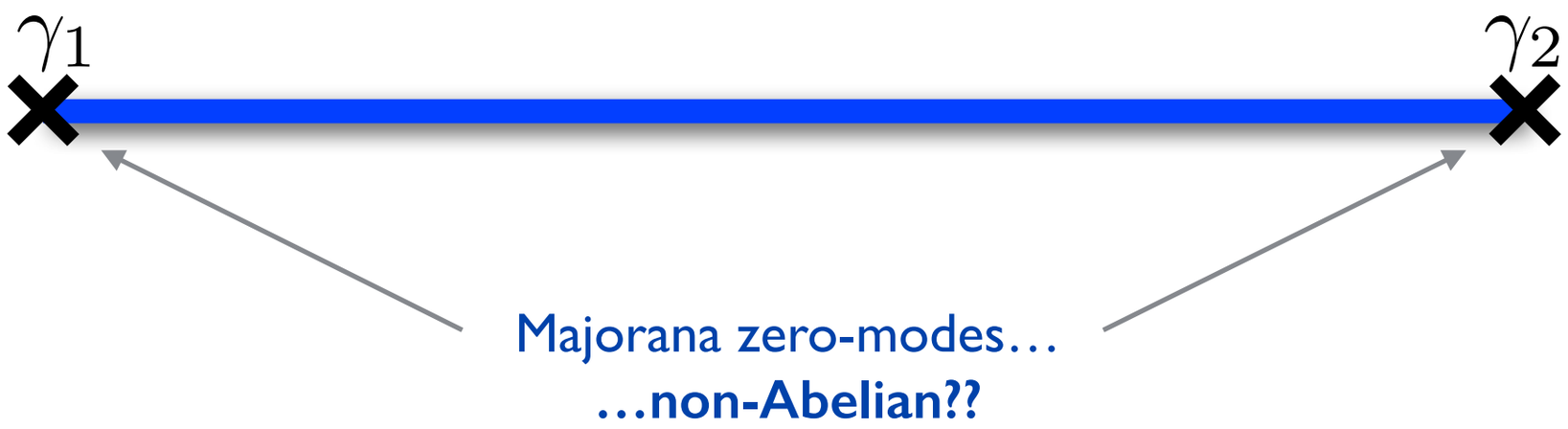

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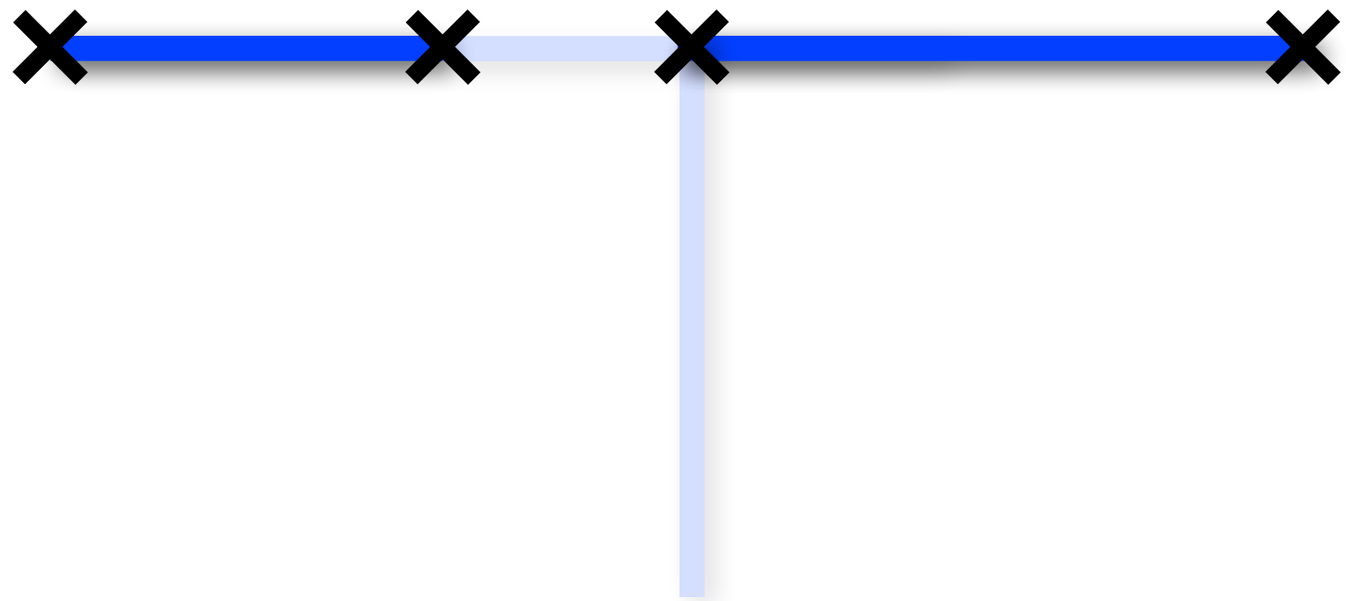

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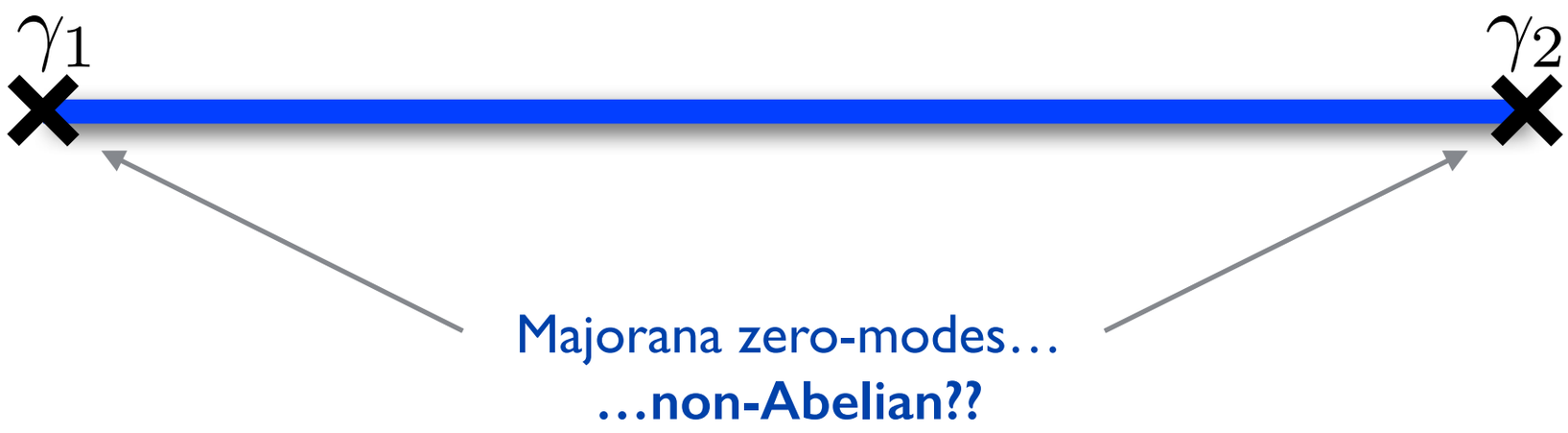

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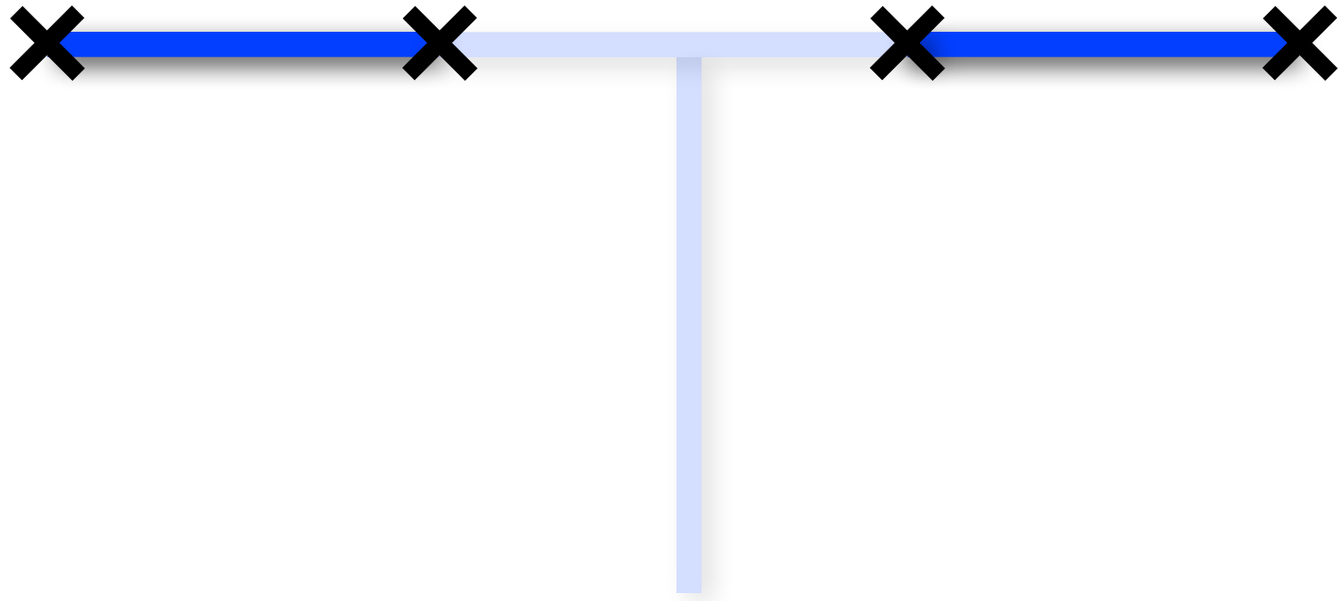

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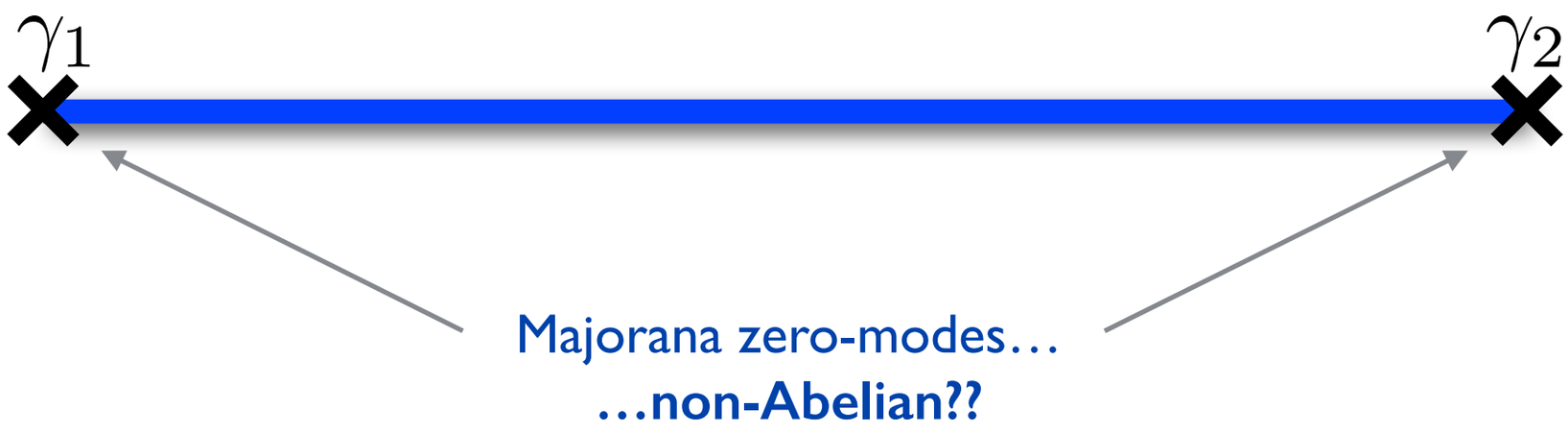
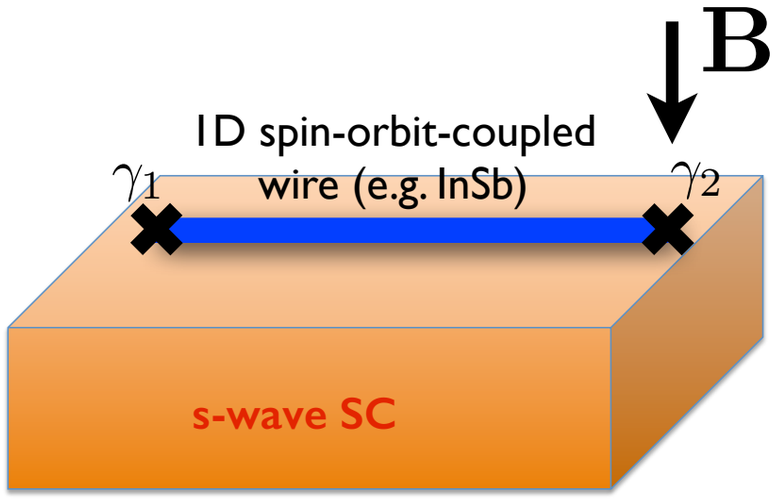
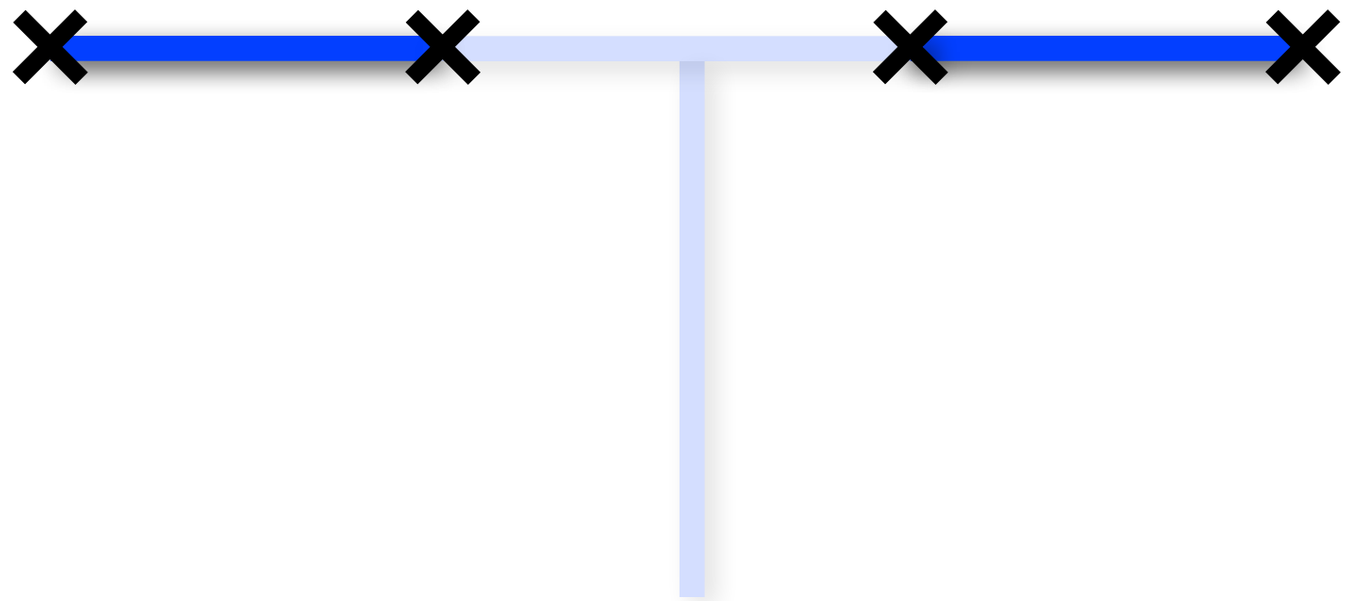


Kitaev (2001)

Designer 1D Majorana platforms



Lutchyn, Sau, Das Sarma (2010);
Oreg, Refael, von Oppen (2010)



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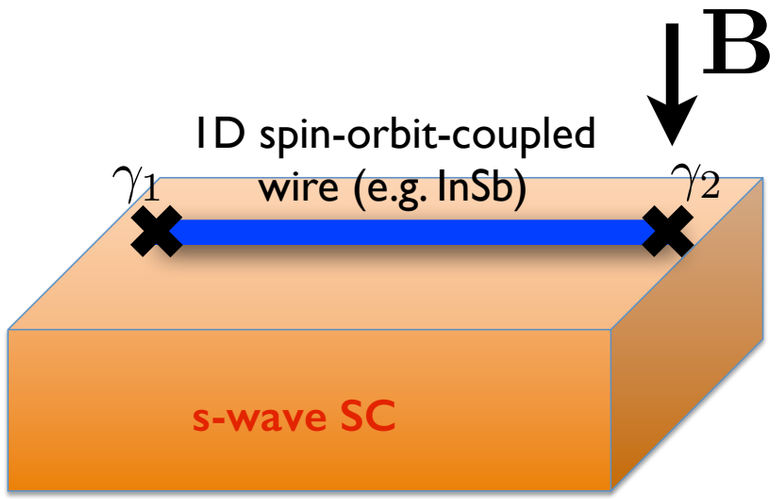

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Oreg, Refael, von Oppen (2010)

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1†]

Evidence of Majorana fermions in an Al – InAs nanowire topological superconductor

Anindya Das*, Yuval Ronen*, Yonatan Most, Yuval Oreg, Moty Heiblum#, and Hadas Shtrikman

Observation of the fractional ac Josephson effect: the signature of Majorana particles

Leonid P. Rokhinson,[1,2,*] Xinyu Liu,[3] and Jacek K. Furdyna[3]

Observation of Majorana Fermions in a Nb-InSb Nanowire-Nb Hybrid Quantum Device

M. T. Deng,[1] C. L. Yu,[1] G. Y. Huang,[1] M. Larsson,[1] P. Caroff,[2] and H. Q. Xu[1,3,*]

Anomalous Modulation of a Zero-Bias Peak in a Hybrid Nanowire-Superconductor Device

A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, and X. Li

Phys. Rev. Lett. **110**, 126406 (2013)

Superconductor-Nanowire Devices from Tunneling to the Multichannel Regime: Zero-Bias Oscillations and Magnetoconductance Crossover

H. O. H. Churchill,[1,2] V. Fatemi,[2] K. Grove-Rasmussen,[3] M. T. Deng,[4] P. Caroff,[4] H. Q. Xu,[4,5] and C. M. Marcus

Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor

(not nanowire setup)

Stevan Nadj-Perge,[1*] Ilya K. Drozdov,[1*] Jian Li,[1*] Hua Chen,[2*] Sangjun Jeon,[1] Jungpil Seo,[1] Allan H. MacDonald,[2] B. Andrei Bernevig,[1] Ali Yazdani[1†]

Spin-resolved Andreev levels and parity crossings in hybrid superconductor–semiconductor nanostructures

Eduardo J. H. Lee[1], Xiaocheng Jiang[2], Manuel Houzet[1], Ramón Aguado[3], Charles M. Lieber[2] and Silvano De Franceschi[1]*

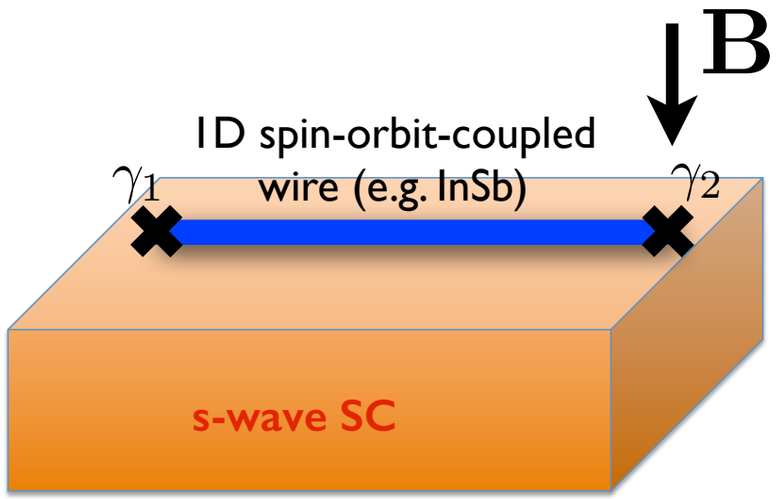
Hard gap in epitaxial semiconductor–superconductor nanowires

W. Chang[1,2], S. M. Albrecht[1], T. S. Jespersen[1], F. Kuemmeth[1], P. Krogstrup[1], J. Nygård[1] and C. M. Marcus[1]*

Parity lifetime of bound states in a proximitized semiconductor nanowire

A. P. Higginbotham,[1,2,*] S. M. Albrecht,[1,*] G. Kiršanskas,[1] W. Chang,[1,2] F. Kuemmeth,[1] P. Krogstrup,[1] T. S. Jespersen,[1] J. Nygård,[1] K. Flensberg,[1] and C. M. Marcus[1]

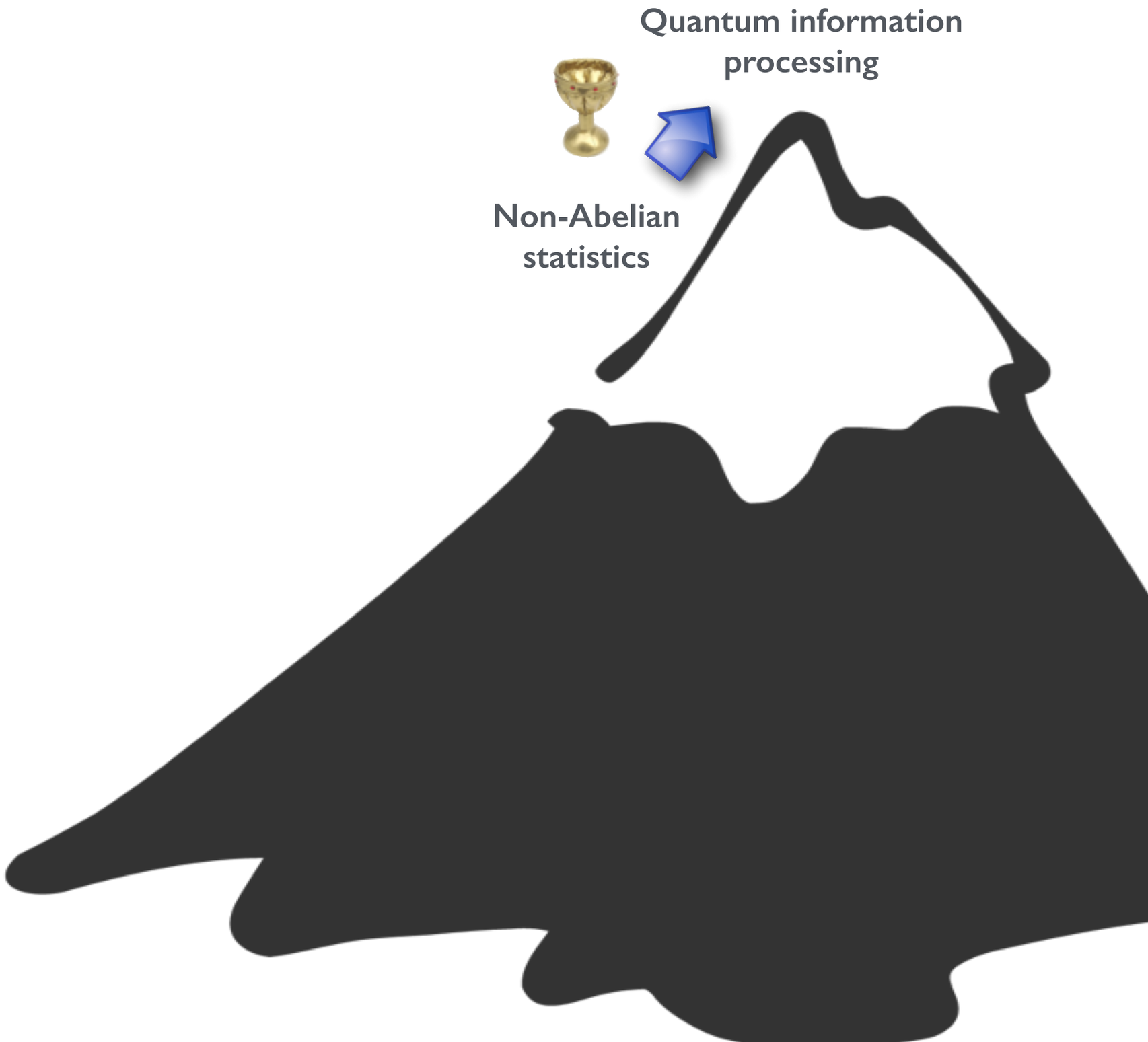
Majorana Milestones



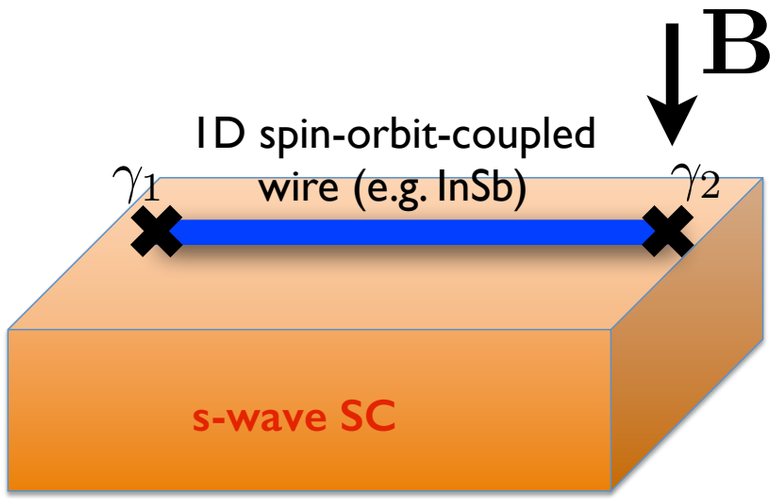
Lutchyn, Sau, Das Sarma (2010);
Oreg, Refael, von Oppen (2010)

Zero-mode detection

(probably yes!)



Majorana Milestones



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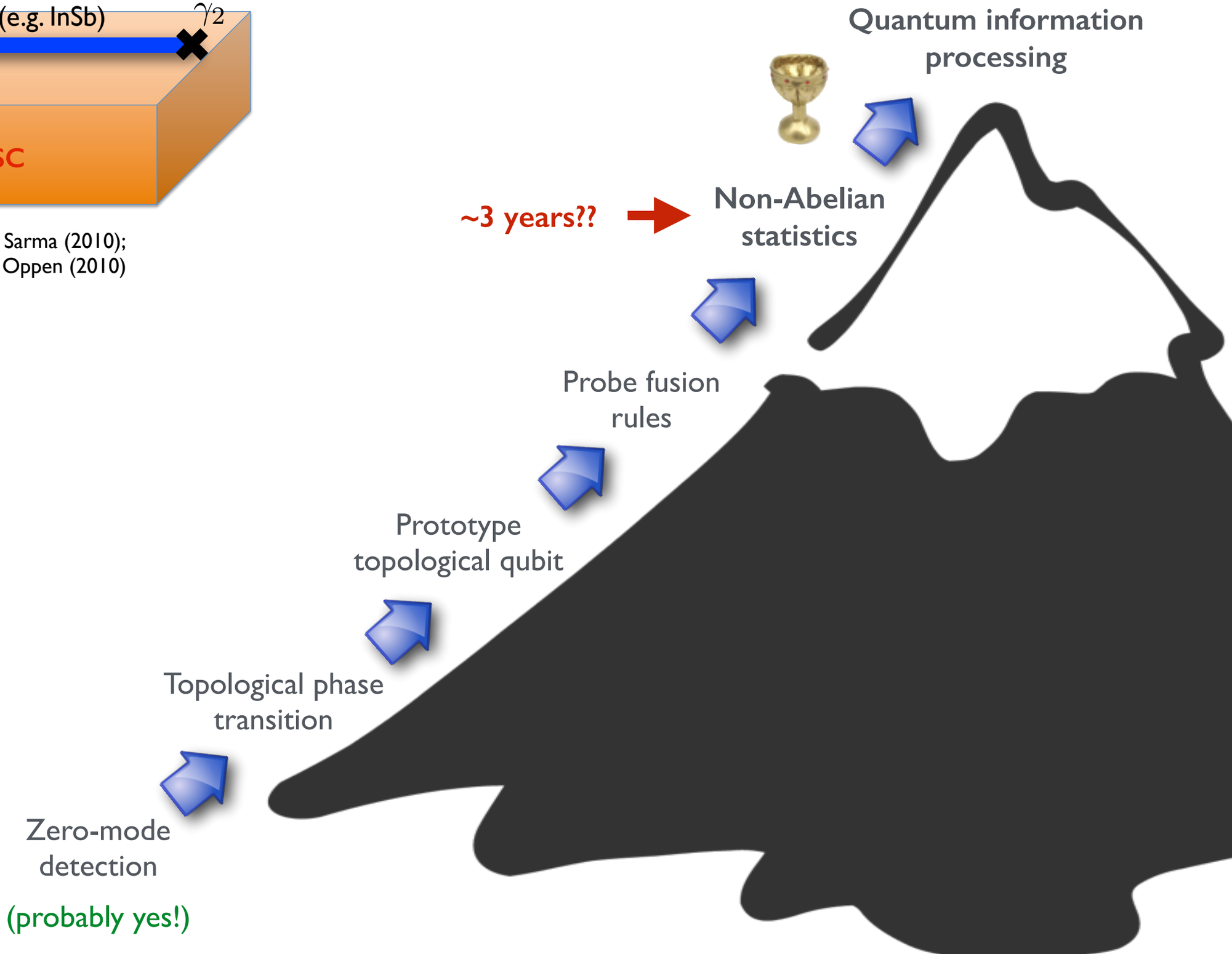
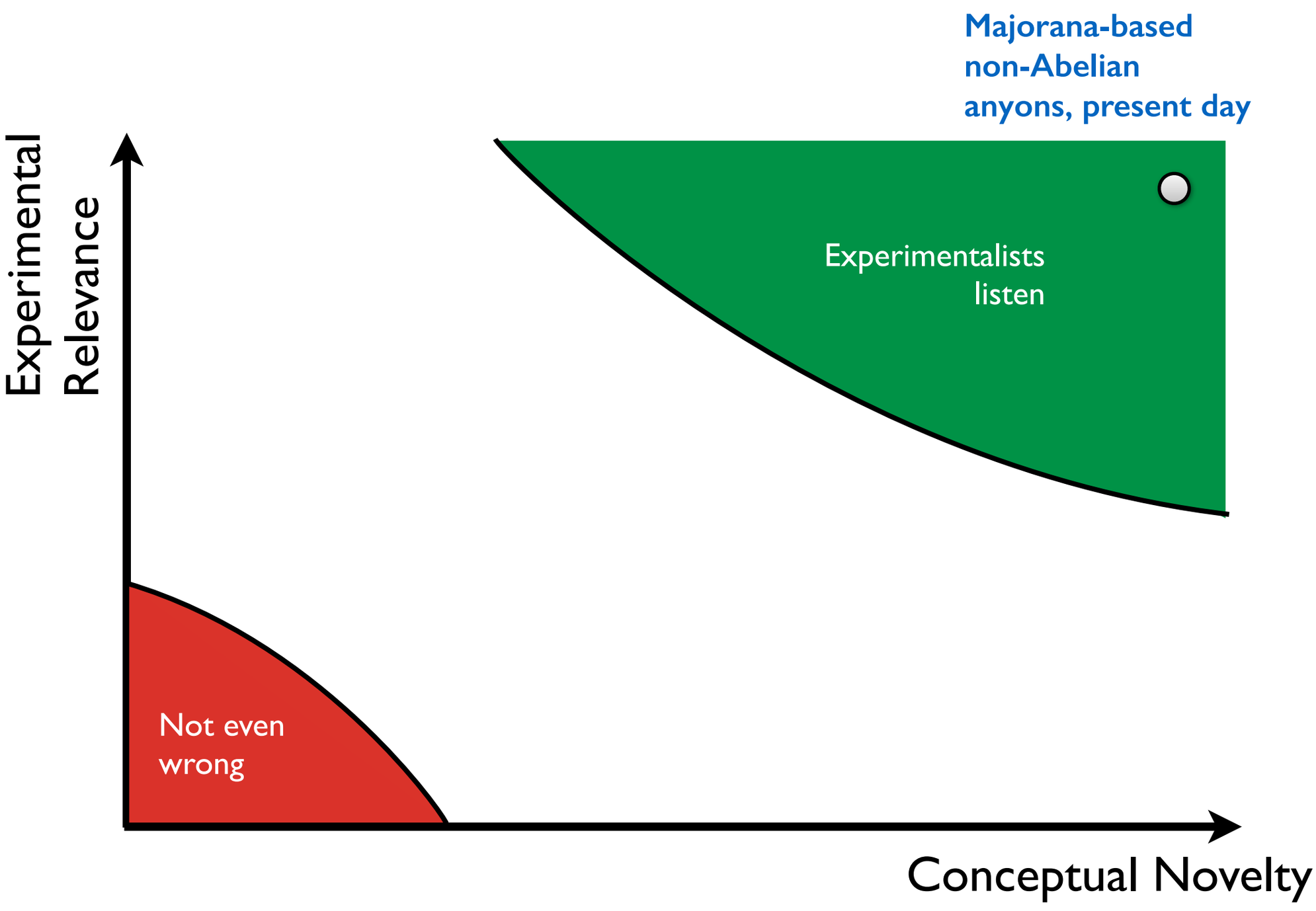
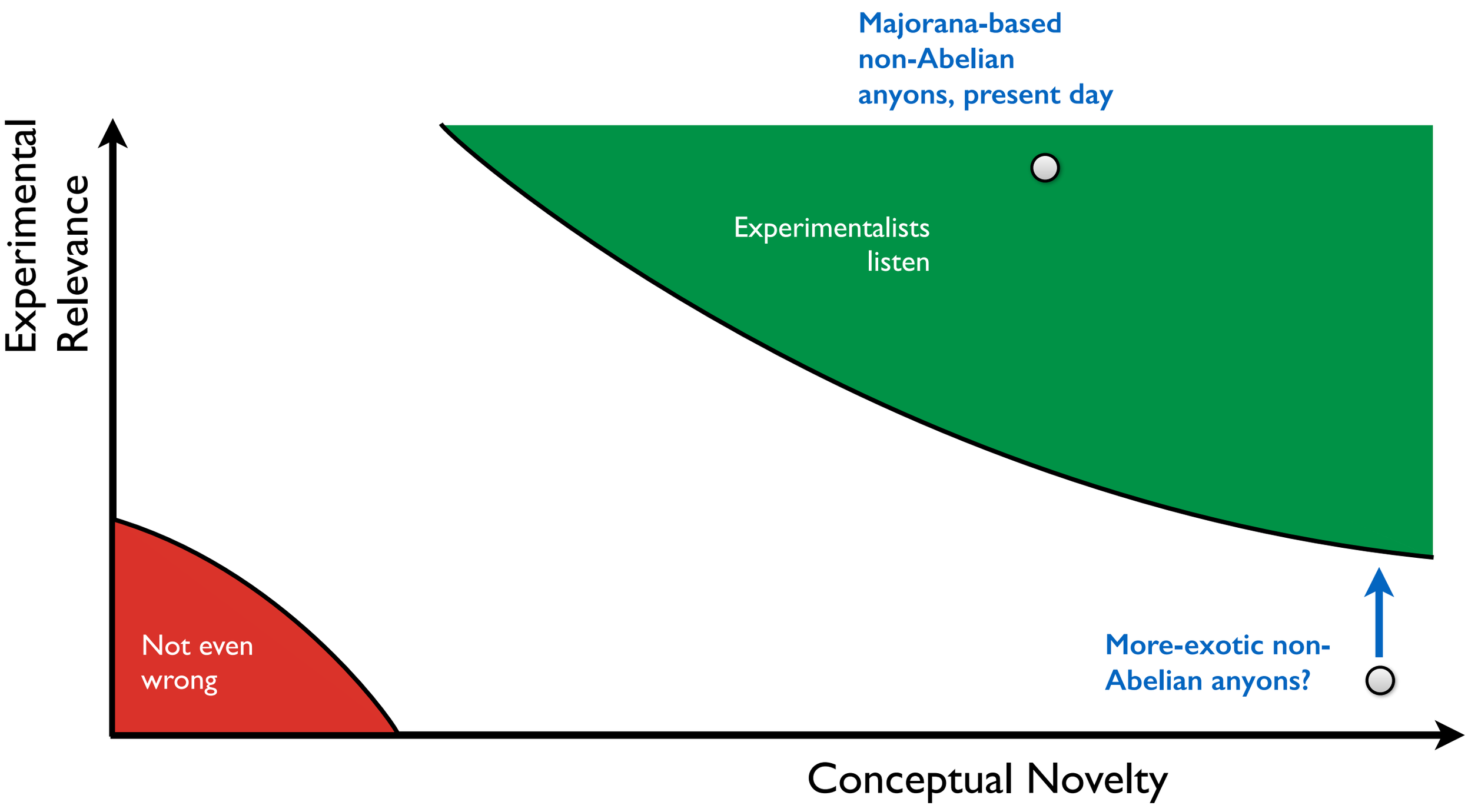


“Fisher plot”


Majorana-based
non-Abelian
anyons, present day
Experimental
Relevance
Experimentalists
listen
Not even
wrong
Conceptual Novelty

“Fisher plot”


Majorana-based
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Experimental
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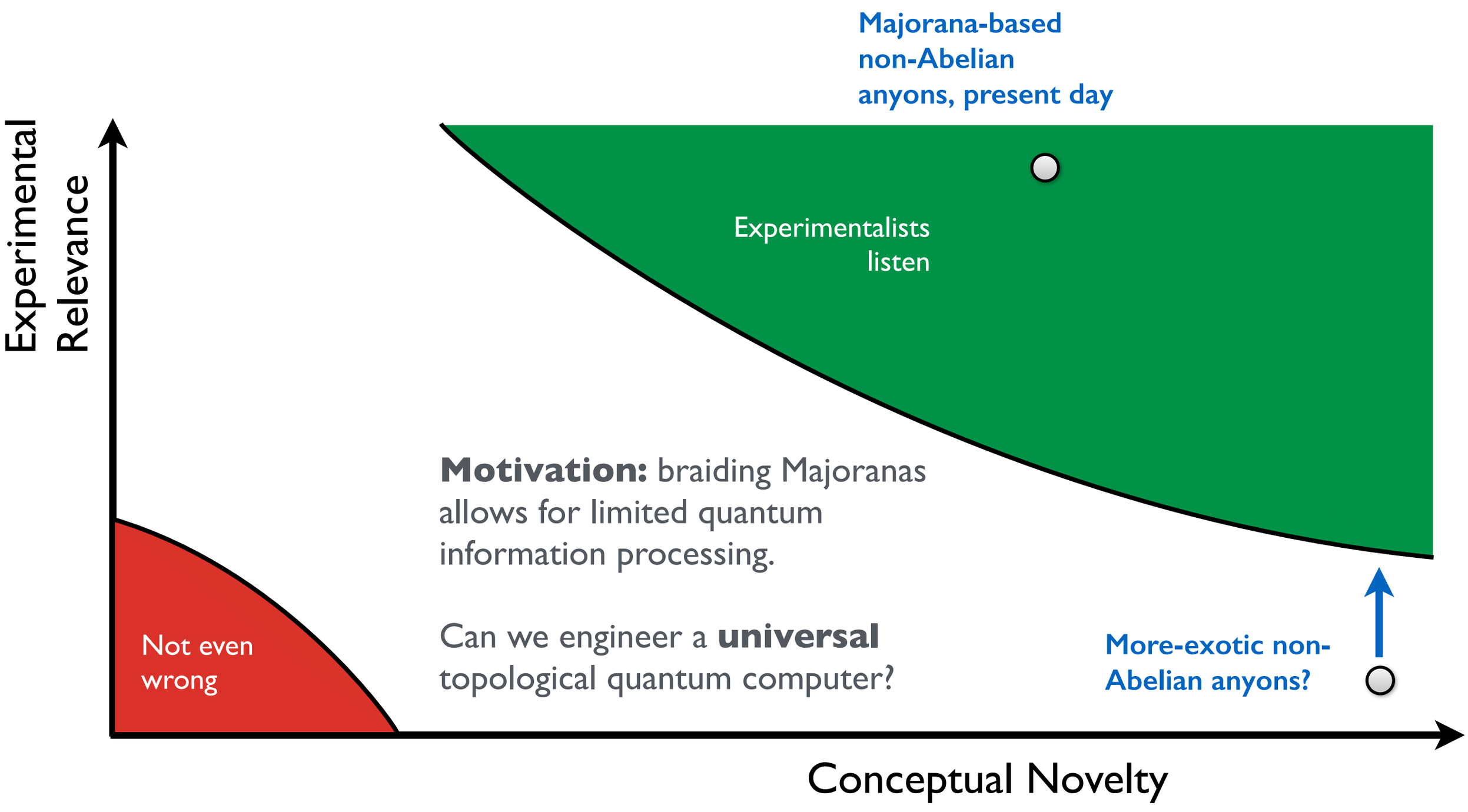
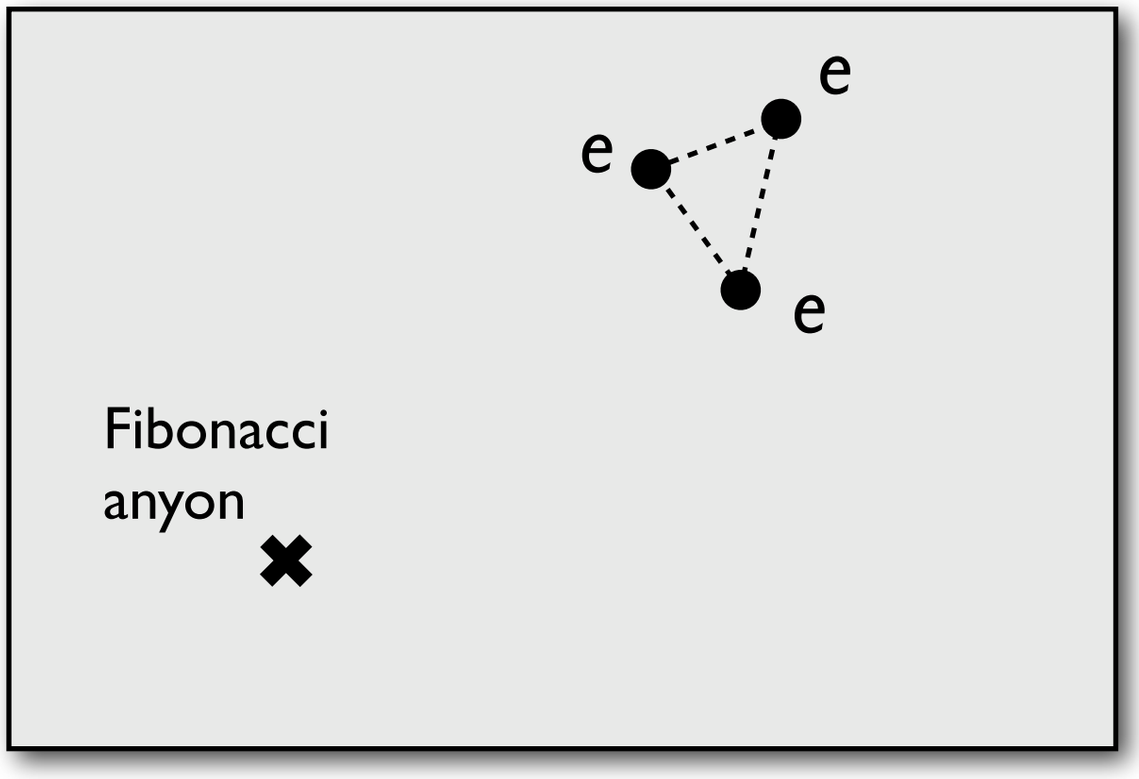
More-exotic non-
Abelian anyons?
Conceptual Novelty

“Fisher plot”

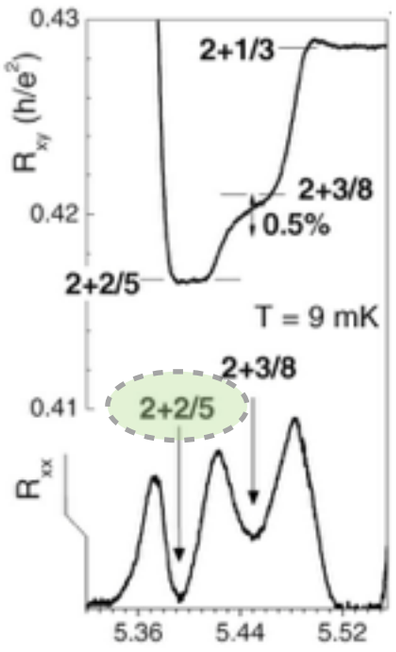

Majorana-based
non-Abelian
anyons, present day
Experimental
Relevance
Experimentalists
listen
Motivation: braiding Majoranas
allows for limited quantum
information processing.
Can we engineer a universal
topological quantum computer?
Not even
wrong
More-exotic non-
Abelian anyons?
Conceptual Novelty

Fibonacci anyons

Z_3 Read-Rezayi state



Read & Rezayi (1999)



Xia et al. (2004)

R. Mong, D. Clarke, JA, N. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, M. Fisher (2014)

M. Stoudenmire, D. Clarke, R. Mong, JA (2015)

Fibonacci anyons

Z_3 Read-Rezayi state

e
e
e

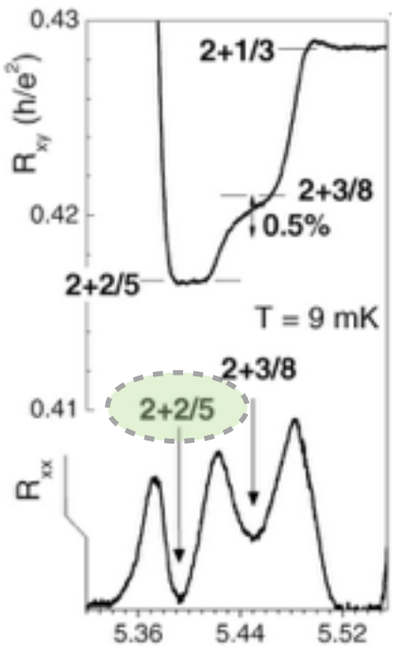
Fibonacci
anyon

Superconducting cousin

"Simple" Abelian quantum Hall state

$\sim$

Read & Rezayi (1999)



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Fibonacci anyons

Z_3 Read-Rezayi state

e
e
e

Fibonacci anyon

Read & Rezayi (1999)

Superconducting cousin

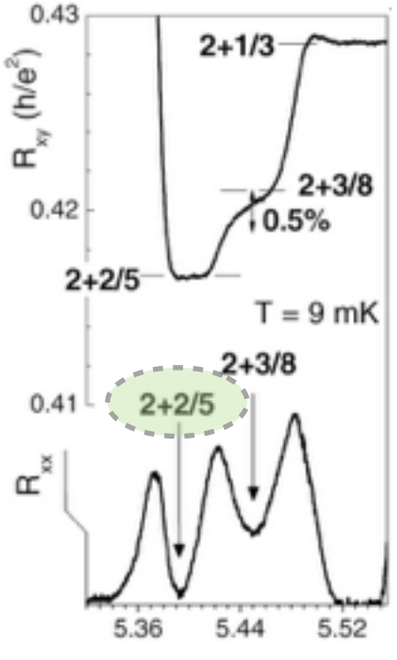
"Simple" Abelian quantum Hall state

2D array of superconducting Islands

2e

R. Mong, D. Clarke, JA, N. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, M. Fisher (2014)

M. Stoudenmire, D. Clarke, R. Mong, JA (2015)



Xia et al. (2004)

Fibonacci anyons

Z_3 Read-Rezayi state

e
e
e

Fibonacci anyon

Read & Rezayi (1999)

Superconducting cousin

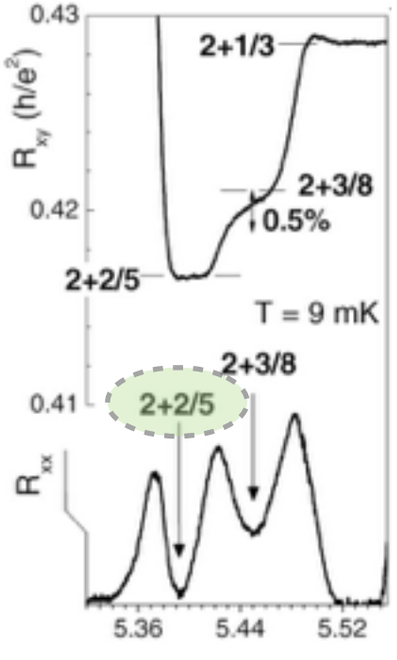
"Simple" Abelian quantum Hall state

2D array of superconducting Islands

2e

Fibonacci anyon

$$2e = 3 \times (2e/3)$$



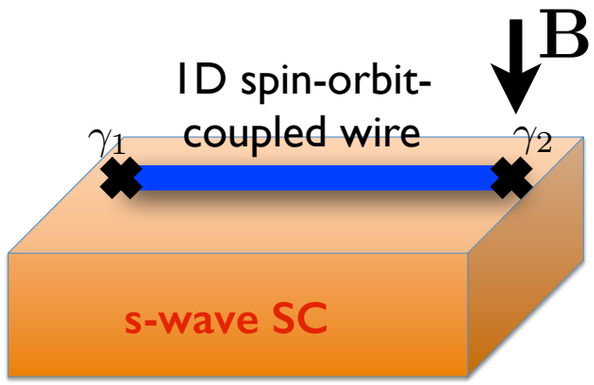
Xia et al. (2004)

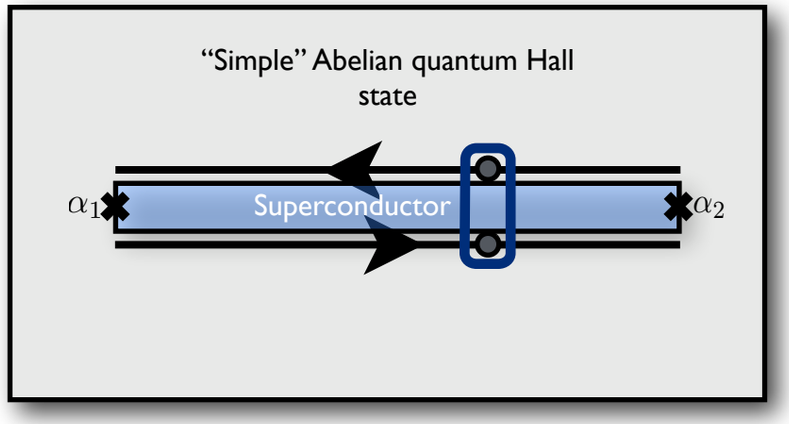
Proof of principle: can combine well-understood systems to create hardware for **universal topological quantum computer!** (but not yet practical)

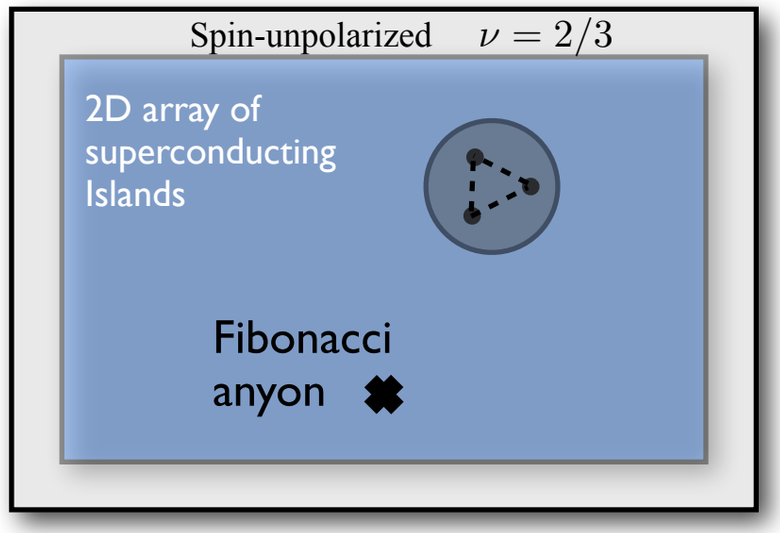
R. Mong, D. Clarke, JA, N. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, M. Fisher (2014)

M. Stoudenmire, D. Clarke, R. Mong, JA (2015)

Many roads to non-Abelian anyons…



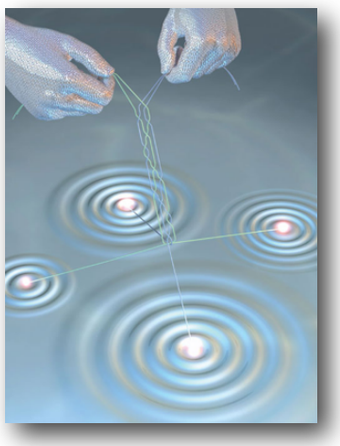




Majorana zero-modes

Parafermion zero-modes

Computational universality!



Non-Abelian statistics, fault-tolerant quantum computation

For a lighthearted overview, see JA & A. Stern, arXiv:1410.0359

Acknowledgments

Dave Aasen (Caltech)
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Adam Jermyn (Caltech)
Liang Jiang (Yale)
Torsten Karzig (Caltech)
Takuya Kitagawa (Harvard)
Shu-Ping Lee (Caltech)
Netanel Lindner (Technion)
Roman Lutchyn (Station Q)
Karen Michaeli (Weizmann)
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Ady Stern (Weizmann)
Miles Stoudenmire (Perimeter)
Felix von Oppen (Berlin)
Amir Yacoby (Harvard)

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CALIFORNIA INSTITUTE OF TECHNOLOGY 1891

Walter Burke Institute
for Theoretical Physics

ALFRED P. SLOAN FOUNDATION 1934

Parafermion zero-modes

Fendley (2012); inspired by Fradkin & Kadanoff

$$\alpha_j \alpha_k = \alpha_k \alpha_j e^{\pm i 2\pi/3} \qquad \alpha_j^\dagger = \alpha_j^2 \qquad \alpha_j^3 = 1$$

Parafermion zero-modes

Fendley (2012); inspired by Fradkin & Kadanoff

$$\underbrace{\alpha_j \alpha_k = \alpha_k \alpha_j e^{\pm i 2\pi/3}} \qquad \alpha_j^\dagger = \alpha_j^2 \qquad \alpha_j^3 = 1$$

Clue: "anyonic" commutation relations…

…make "wire" in system with Abelian anyons?

Parafermion zero-modes

Fendley (2012); inspired by Fradkin & Kadanoff

$$\alpha_j \alpha_k = \alpha_k \alpha_j e^{\pm i 2\pi/3} \qquad \alpha_j^\dagger = \alpha_j^2 \qquad \alpha_j^3 = 1$$

"Simple" Abelian quantum Hall state

e.g., $\nu = 2/3$

Clarke, JA, Shtengel (2014); Lindner, Berg, Stern, Refael (2013); Cheng (2013); Qi et al. (2013-14)

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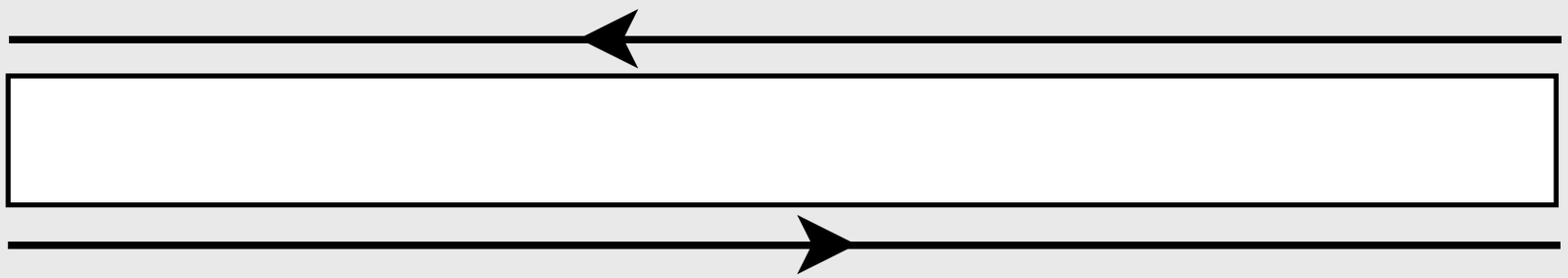

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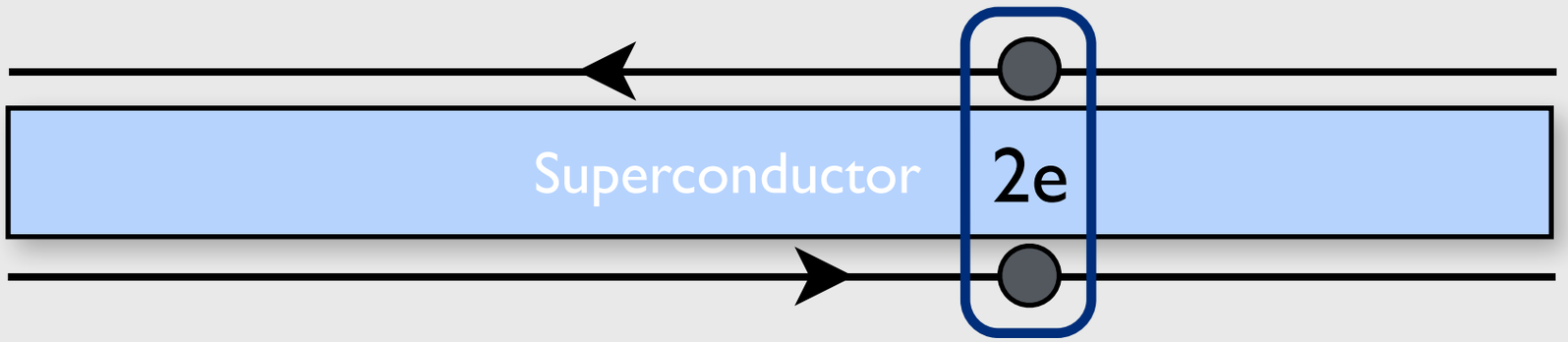

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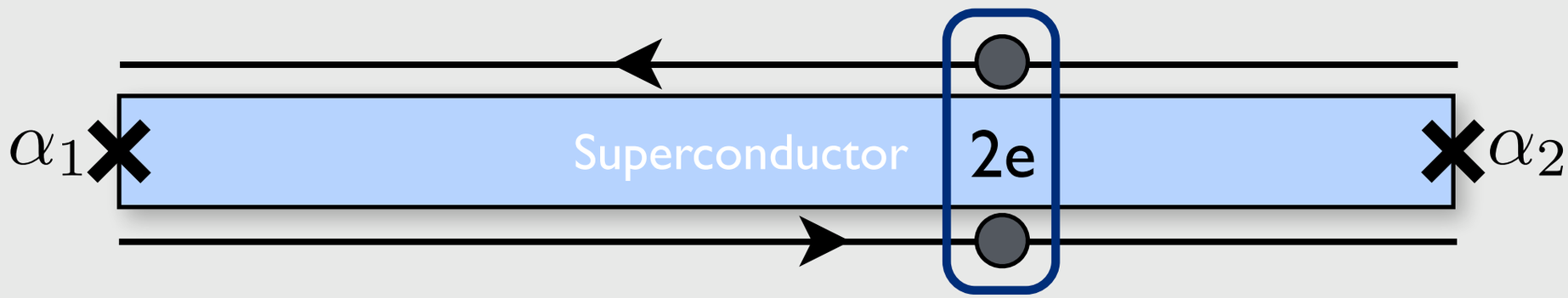

Fractionalized cousin of Kitaev's 1D "spinless" superconductor!

Parafermion braiding gives additional gate for free, but still not universal…

Clarke, JA, Shtengel (2014); Lindner, Berg, Stern, Refael (2013); Cheng (2013); Qi et al. (2013-14)